An editorial history of Newton's *regulae philosophandi**

Una historia editorial de las *regulae philosophandi* de Newton

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Fecha de recepción: 18 de noviembre de 2014
Fecha de aprobación: 1º de diciembre de 2014
DOI: 10.17533/udea.ef.n51a09

Resumen. *El presente trabajo, basado en la obra publicada e inédita de Isaac Newton, proporciona una historia editorial de sus famosas regulae philosophandi.*

Palabras clave: *Isaac Newton, principia mathematica, regulae philosophandi, metodología científica*

Abstract. *In the paper at hand, I provide an editorial history of Isaac Newton's famous regulae philosophandi on the basis of extant manuscript material.*

Keywords: *Isaac Newton, principia mathematica, regulae philosophandi, scientific methodology*

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* The research that resulted in this article was funded by the Special Research Fund of the Vrije Universiteit Brussel under the form of a research professorship. For manuscripts pertaining to the Portsmouth Collection I have consulted their digital reproductions at cudl.lib.cam.ac.uk, their diplomatic transcriptions at www.isaacnewton.sussex.ac.uk, and the originals at Cambridge University Library. I am indebted to the Provosts and Syndics of Cambridge University Library for permission to quote from the Portsmouth and Macclesfield Collection. In this essay, I will expand my analysis of Newton's *regulae philosophandi* as provided in Ducheyne (2012). Part of this essay was presented at the international conference 'A great variety of admirable discoverys': *Newton's Principia* in the Age of Enlightenment which took place on 11-13 December 2013 at the Royal Society, London. I am indebted to its audience for useful feedback. I am also indebted to the anonymous referee for several useful remarks. I alone assume responsibility for any remaining mistakes. Conventions for transcriptions: arrows pointing downwards (↓…↓) indicate that the text between them was added above the line; arrows pointing upwards (↑…↑) indicate that the text between them was added below the line; pointy brackets (<…>) indicate that the text between them is an editorial comment, and finally braces ({…}) indicate that the text between them has been supplemented in case certain words are missing from the original or highly unreadable.

Como citar este artículo:
MLA: Ducheyne, S. "An editorial history of Newton's regulae philosophandi". Estudios de Filosofía 51 (2015): 143-164.
APA: Ducheyne, S. (2015). An editorial history of Newton's regulae philosophandi. Estudios de Filosofía. (51), 143-164.
Chicago: Steffen Ducheyne. "An editorial history of Newton's regulae philosophandi," Estudios de Filosofía n.º 51 (2015): 143-164.

Richard P. Feynman once described Newton's theory of universal gravitation as "probably one of the most far-reaching generalizations of the human mind" (2005, disc 1: 0:28-0:33). The *regulae philosophandi*, which got their name in the second edition of the *Principia*,[1] were vital in the establishment of Newton's bold generalization. In the additions and corrections to the first edition of the *Principia*, Newton recorded that in experimental philosophy propositions are deduced from phenomena and rendered general by the *regulae philosophandi* (CUL Add. Ms. 3965: 544^r).[2] The function of the *regulae philosophandi*, as he was well aware, is to provide justification for the ampliative conclusions that occur in Book III of the *Principia*. Correspondingly, if one seeks an understanding of the way in which ampliative conclusions are established in Newton's natural-philosophical methodology, one should come to terms with the development of the *regulae philosophandi*.

In the paper at hand, I will provide an editorial history of Newton's four rules of philosophizing. In other words, I will document – both in Newton's published as well as in his unpublished work – the development of the rules of philosophizing from their earliest inception to the final form under which they appeared in the third edition of the *Principia* (1726). It is not my primary concern to analyze the methodological significance of these rules in full dept.[3] Instead, in the next section I shall attempt to unearth the meaning of the *regulae* and to draw attention to the specific context out of which each of them emerged. In order to do so, I shall take into account all of Newton's published and unpublished writings on the *regulae*. In the third section, I shall briefly comment upon what the *regulae* tell us about Newton's methodological development.

1 In the first edition of the *Principia* (1687), Rules I and II were labeled as hypotheses. In later editions Newton referred to them as *'regulae philosophandi'*. Rules III and IV made their first appearance in the second and third edition of the *Principia* (1713; 1726), respectively. In a memorandum composed on 21 July 1706, David Gregory recorded that Newton "now calls" Hypotheses I-II in the first edition of the *Principia* "*Regulae Philosophandi*" (Hiscock (1937). p. 36).

2 Very useful information on Newton's *regulae philosophandi* is to be found in Koyré (1965); Cohen (1966), (1971). pp. 23-26, pp. 259-62, and (1999). pp. 198-204; and McGuire (1995). Chapter 2 and 6. I am indebted to all of these studies. For recent studies of the *regulae*, Mamiani (2004) and Spencer (2004).

3 See Ducheyne (2012). pp. 109-120 for my own take on the methodological significance of Newton's *regulae*.

The *regulae philosophandi*

Rules I and II

In the first edition of the *Principia*, Rules I and II were labelled 'Hypothesis I' and 'Hypothesis II', respectively (Newton, 1999: 794, note a). In the second and third edition of the *Principia*, Rule I states in the Cohen-Whitman translation:

> *No more causes of natural things should be admitted than are both true and sufficient to explain their phenomena.* (Newton, 1999: 794; Newton, 1713: 357 and 1726: 387)[4]

In the first edition, Rule I reads "sufficiunt" instead of the correct "sufficiant", which just as the other subjunctive "sint" expresses purpose in combination with "quam." Its commentary reads: "For nature is simple and does not abound in superfluous causes of things [rerum causis superfluis non luxuriat]" (Newton, 1687: 402). The commentary to Rule I lacks the following addition that appeared in the second and third edition of the *Principia*: "As the philosophers say: Nature does nothing in vain, and more causes are in vain when fewer suffice. For nature is simple and does not indulge in the luxury of superfluous causes" (Newton, 1999: 794).[5] In the third edition of the *Principia*, Rule II states:

> *Therefore, the causes assigned to natural effects of the same kind must be, so far as possible, the same*. (Newton, 1999: 795)[6]

In the first and second edition, Rule II states:

> *Therefore, causes of natural effects of the same kind are the same* [*eædem sunt causæ*]. (Newton, 1999: 795; 1687: 402; 1713: 357)

The commentary to Rule II is identical in all editions:

> Examples are the cause of respiration in man and beast, or the falling of stones in Europe and America, or of the light of a kitchen fire and the sun, or of the reflection of light on earth and the planets. (Newton, 1999: 795)

Given that Newton wrote "ideoque" at the beginning of Rule II, the second ruled is to be conceived as a consequence of Rule I.

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4 Translation of: "*Causas rerum naturalium non plures admitti debere, quam quæ & veræ sint & earum phænomenis explicandis sufficiant.*" (Koyré, Cohen & Anne Whitman (1972), II. p. 550).

5 Newton inserted this addition in one of his private copies of the first edition of the *Principia* (CUL Adv.b.39.1. p. 402).

6 Translation of: "*Ideoque effectuum naturalium ejusdem generis eædem assignandæ sunt causæ, quatenus fieri potest.*" (Koyré, Cohen & Anne Whitman (1972), II. p. 550). This text corresponds exactly to Newton's corrected version in one of his copies of the second edition of the *Principia* (WL NQ.16.196. 357). The corresponding manuscript material is to be found on CUL Add. Ms. 3965. 419[r] and 519[r].

Newton's corrections and additions to the second edition of the *Principia* contain a variant of Rule II, which indicates that the causes referred to in that rule are to be conceived of as proximate causes (CUL Add. Ms. 3965: 419^r).[7] Although its corrected version corresponds to Rule II as published in the third edition of the *Principia*, the surrounding text, which Newton deleted, is also worth looking at. Newton's first attempt reads: "[t]herefore the causes of natural effects of the same kind are to be assumed as being the same," a statement which he then corrected into "[t]herefore the proximate causes to be assigned to natural effects of the same kind are the same." The remainder of the text contains a proviso that is absent from the published version. Newton needed three attempts to arrive at "unless somehow a diversity from phenomena is disclosed." At the end of this fragment, Newton added the words "so far as possible [quatenus fieri potest]." By mobilizing Rules I and II in Proposition IV and in Proposition V (and their *scholia*), Book III (Newton, 1999: 804-805, 806), Newton was entitled to claim that the inverse-square centripetal forces drawing the primary planets to the sun and those drawing the secondary planets to the earth, Jupiter or Saturn are instances of the same cause.

In the *Principia* Newton conceived of centripetal forces as the proximate causes of the motion of terrestrial and celestial bodies. In what sense should the instances of centripetal forces that Newton inferred in Book III on the basis of Propositions I-III, Book I be understood as causes? In Propositions I and II, Book I Newton established that a centripetal force by which a body is drawn towards an unmoving center of force is directed exactly to this center, if and only of, that body describes equal areas, which lie in a fixed plane, in exactly equal times (Newton, 1999: 444-448).[8] Both directions are based on a deduction from the first law of motion, which states that "[*e*]*very body perseveres in its state of being at rest or of moving uniformly straight forward, except insofar as it is compelled to change its state by forces impressed*" (Newton, 1999: 416). Law I stipulates the conditions

7 My translation of: "Ideoque Effectuum naturalium ejusdem generis eædem ~~assumendæ~~ ↓assignandæ↓ sunt causæ ~~nisi quatenus diversitas~~ ↓~~[proximæ nisi [forte diversitas aliqua↓ ex phænomenis patefacta sit hæ causæ phænomenis explicandis sufficiant.], nisi diversitas ↓aliqua↓ ex phænomenis patefacta sit.]~~ quatenus fieri potest."

8 In Proposition III, Book I Newton furthermore established that the overall centripetal force by which a body is drawn towards a second body is directed to this body *as most closely as possible* (quam proxime), if and only if, that body describes equal areas in equal times *as most closely as possible* (Newton (1999). pp. 448-449). The methodological significance of the *quam proxime* counterparts of Propositions I-II, Book I has been highlighted in Smith (2002), Harper (2012). pp. 113-114, and Ducheyne (2012). pp. 82-83, pp. 87-89, pp. 103-104.

under which inertial motion will occur, namely if no impressed force (vis impressa), i.e. according to Definition IV, "*the action which exerted on a body to change its state either of resting or of moving uniformly straight forward*" (Newton, 1999: 405),[9] is acting on a body. Law I, in other words, informs us that if a body describes non-inertial motion, an impressed force is acting on that body. The centripetal forces which Newton inferred in Book III are causal in the sense that their derivation is based on the specific causal intuition that underlies the first law of motion, namely *C* is a counterfactual-nomological cause of *E*, if and only if, there is a law that stipulates that if *C* had not occurred, then *E* would not have occurred (Ducheyne, 2012: 34-35). My reading of what it meants for Newton for a centripetal force to be a cause of motion is not only explanatory of Newton's frequent use of causal terms in the *Principia*, but also with his denial that he had established the full cause of gravitational effects: by relying on this counterfactual-nomological intuition Newton could state that he had inferred the proximate causes of gravitational effects, given the laws of motion, without having to address the (full) cause of gravity.

As we have seen, Rule I applies only to causes that are true and sufficient to explain their phenomena.[10] In the previous paragraph, I have drawn attention to Propositions I-II, Book I. Given the causal reading I have proposed, we might say that in Propositions I-II, Book I Newton established that a centripetal force directed exactly towards a center of force is, given the laws of motion, the necessary and sufficient cause of Kepler's area rule to hold exactly.[11] More precisely, Proposition I establishes the sufficient direction which shows that, if a centripetal force is acting on a body, it will describe Kepler's area Rule and Proposition II the necessary direction which shows that, if a body describes Kepler's area law, it is drawn by a centripetal force. In his sophisticated natural-philosophical methodology Newton insisted that both directions are to be demonstrated, because he was critical of founding natural philosophy on sufficient causes only. The direction covered in Proposition I agrees to what Newton called causes that are sufficient to explain their phenomena in Rule I; the direction covered in Proposition II covers what Newton referred to as causes that are true. The up-shot of this is that Rule I prescribes keeping only necessary and sufficient causes to a minimum.[12]

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9 Newton distinguished between three sources of impressed force: percussion, pressure or centripetal force.

10 Cf. Smith (2002). p. 160.

11 We have also seen that in Proposition III, Book I Newton established that an overall centripetal force directed *quam proxime* towards a second body is, given the laws of motion, a necessary and sufficient cause for Kepler's area rule to hold *quam proxime*.

12 Note that Newton's argument for the heterogeneity of white light cannot be justified by Rule II. As we have seen, Rule II licenses the identification of instances of causes of the same kind which have been

Rule III

In Rule III, which was added in the second edition of the *Principia*, Newton explicated the conditions under which certain qualities are to be taken as universal qualities.[13] In the second and third edition of the *Principia*, Rule III states:

> *Those qualities of bodies that cannot be intended and remitted [i.e., qualities that cannot be increased and diminished]*[14] *and that belong to all bodies on which experiments can be made should be taken as qualities of all bodies universally*. (Newton, 1999: 795)[15]

One of the earliest formulations of Rule III is to be found in a list of corrections and additions to the first edition of the *Principia* which Newton composed in the early 1690s. During this period, Newton was making plans for a new edition of the *Principia* (Westfall, 1980: 506-512) – a project that ultimately materialized more than twenty years later. Nicolas Fatio De Duillier and David Gregory after him shortly acted as prospective editors of a new edition of Newton's *magnum opus* in the early 1690s (Cohen, 1971: 177-184, 189-198). At some point, Newton also seems to have considered adding the so-called 'Classical Scholia', in which he reported on the views of the ancients regarding matter, gravity and motion, to the new edition.[16] In his corrections to the first edition of the *Principia* Newton introduced a hypothesis that would become Rule III. It reads:

> Hypothesis III. Qualities of bodies that cannot be intended and remitted and that belong to all bodies upon which experiments are allowed to be made are qualities of all bodies universally. The same is to be understood of qualities of all bodies of the same kind. [This rule] is seen to be the fundament of all philosophy. For otherwise it is not allowed to derive the qualities of insensible bodies from the qualities of sensible ones. (CUL Add. Ms. 3965: 266^{r})[17]

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shown to be necessary and sufficient to explain phenomena of the same kind. In his argument for the heterogeneity of white light Newton instead relied on the supposition that the light before and after refraction is "of the same Temper and Constitution" (Newton (1979). p. 55). The fact that Newton never relied on Rule II to justify his argument for the heterogeneity of white light is highly significant.

13 Newton mobilized explicitly Rule III in Corollary 2 to Proposition VI, Book III (Newton (1713). p. 368 and (1726). p. 402). McGuire has drawn considerable attention to the draft versions of Rule III (McGuire (1995). pp. 69-72).

14 This is an addition inserted by the translators. In what follows, I shall argue that this addition is quite problematic.

15 Translation of: "*Qualitates corporum quæ intendi & remitti nequeunt, quæque corporibus omnibus competunt in quibus experimenta instituere licet, pro qualitatibus corporum universorum habendæ sunt.*" (Koyré, Cohen & Whitman (1972), II. p. 552).

16 See McGuire & Rattansi (1966) and Casini (1984) for contextualization. The Classical Scholia are transcribed in Schüller (2001).

17 My translation of: "Hypoth. III. Qualitates corporum quæ intendi et remitti nequeunt quæque corporibus omnibus competunt in quibus experimenta instituere licet sunt qualitates corporum universorum. Idem intelligendum est de qualitatibus corporum omnium ejusdem generis. Fundamentum ↓esse↓ videtur Philosophiæ totius. Neque enim aliter ↓qualitates ↓corporum↓ insensibilium↓ a qualitatibus sensibilium

Note that Newton stipulated the conditions under which certain qualities "*are* qualities of all bodies universally." In the published versions of Rule III, Newton reformulated this rather ontological claim into an epistemological one, namely "*are to be taken as* qualities of all bodies universally." In the hypothesis that immediately followed, which is hard to decipher, Newton provided the conditions under which "kinds of things" (*genera rerum*) – by which he probably referred to kinds of forces, such as magnetism and gravity[18] – "*are to be taken* [habendæ sunt]" as new ones (CUL Add. Ms. 3965: 266[r]). This shows that although Newton emphasized the provisional status of this particular hypothesis, he did not do so in the precursor of Rule III. The hypothesis thereafter corresponds to Hypothesis III in the first edition of the *Principia*, which states that "every body can be transformed into a body of any other kind and successively assume all intermediate degrees of qualities" (CUL Add. Ms. 3965: 266[r]).[19] Although one might be tempted to read Hypothesis III as a reference to Newton's belief in alchemical transmutation, it actually concerns the physical reconfiguration of parts of bodies,[20] which is clear from the context in which Newton mobilized Hypothesis III, namely in Corollary 2 to Proposition VI, Book III in the first edition of the *Principia*:

> Therefore all bodies universally that are on or near the surface of the earth are heavy [or gravitate] toward the earth, and the weights of all bodies that are equally distant from the center of the earth are as the quantities of matter in them. For if the aether or any other body whatever either were entirely devoid of gravity or gravitated less in proportion to the quantity of its matter, then, since it does not differ from other bodies except in the form of its matter [forma materiæ], it could by a change of its form be changed by degrees [per mutationem formæ gradatim transmutari[21]] into a body of the same condition as those

~~<illegible word> qualitates insensibilium~~ derivare licet." (cf. CUL Add. Ms. 4005. f. 81[v]). Although Newton reiterated in the commentary to Rule III that it is the fundament of all natural philosophy (Newton (1999). p. 115), he remained silent about the transductive significance of Rule III.

18 See Corollary 5 to Proposition VI, Book III (Newton (1999). p. 810).

19 My translation of "Corpus omne in alterius cujuscunque generis corpus transformari posse, & qualitatum gradus omnes intermedios successivè induere." This formulation is identical to the one in Newton (1687). p. 402. In the same draft, Newton added the sentence "Peripateticorum et Cartesianorum est Hypothesis & contra eorum præjudica solummodo dirigitur." after Hypothesis III (CUL Add. Ms. 3965. f. 266[r]). On Hypothesis III, see Dobbs (1975). pp. 199-204; (1982) and, (1991). pp. 23-24. Although Hypothesis III was deleted in subsequent editions of the *Principia*, it remained central in Newton's argumentation for Corollary 2 to Proposition VI, Book III.

20 Dobbs did not systematically distinguish between transmutation and transformation (e.g., Dobbs (1991). p. 23). Hereby I do not wish to deny per se that Hypothesis III might have an alchemical origin. What I deny, however, is that Newton's usage of Hypothesis III in the *Principia* points to *qualitative* changes in bodies. Compare McGuire (1995). pp. 267-269.

21 By consistently using the infinitive 'transmutari' in all editions of the *Principia*, Newton contributed significantly to Dobbs' misunderstanding which I have brought to the fore in the previous footnote (Newton (1687). p. 411; (1713). p. 368; and, (1726). p. 402). However, if we look at the way in which Hypothesis III is used, it is clear that this hypothesis concerns the reconfiguration of parts of bodies into

> that gravitate the most in proportion to the quantity of their matter (by hyp. 3), and, on the other hand, the heaviest bodies, through taking on by degrees [gradatim induendo] the form of the other body, could by degrees lose their gravity [gravitatem suam gradatim amittere]. And accordingly the weights would depend on the forms of bodies and could be altered with the forms, contrary to what has been proved in corol. 1. (Newton, 1999: 809, note a; 1687: 410–11)

The hypotheses surrounding the above formulation of Rule III offer insight into the context in which it was originally formulated. In CUL Add. Ms. 3965, 266^r Newton was concerned with the relevancy of Rule III for transductive inferences – without thereby entirely reducing it to a transductive inference rule. Typically, when making transductive inferences, we reason from the qualities of observable bodies to qualities of unobservable bodies. In the suppressed preface and conclusion intended for the first edition of the *Principia*, Newton drew considerable attention to the analogy between the laws that govern the motions of great bodies and those that govern the motions of small bodies (Hall & Hall, 1978: 304, 321, 332). In identical vein, in the preface to the first edition of the *Principia*, Newton uttered wishfully: "If only we could derive the other phenomena of nature from mechanical principles by the same kind of reasoning!" (Newton, 1999: 382). It seems, moreover, that the importance of transductive inferences became very important to Newton in the early 1690s, for in the outline of a projected "ffourth book concerning the nature of light & y^e power of bodies to refract & reflect it" (CUL Add. Ms. 3970: 337^r) which seems to have been finished in August 1691 (Shapiro, 1993: 144-147), he sketched the basics of an atomistic theory of light which was based on the analogy between the motions of great bodies and the motions of the small bodies of light.[22] In this very draft material, Newton wrote as follows:

> As all the great motions in the world depend upon a certain kind of force (~~vulgarly called gravity)~~ w^{ch} in this earth we call gravity) whereby great bodies attract one another at great distances: so all the ~~minute~~ ↓little↓ motions in y^e world depend upon certain kinds of forces whereby minute bodies attract or dispell one another at little distances. How ~~all the great motions <illegible word> are regulated by the gravity of ↓which↓ great bodies ↓have↓ towards one another I shewed at large in my ↓Philosophiæ↓ Principia mathematica~~ ↓the great bodies of y^e earth Sun Moon & Planets gravitate towards one another what↓ are y^e laws ~~of~~ & quantities of their gravitating forces at all distance from them & how ~~the~~ all y^e motions of those bodies are regulated by those their gravities I shewed in my ~~Philosophiæ naturalis Principia mathematica by such a convincing ↓mathematical↓ way of arguing as has given satisfaction procured the assent of <illegible words> ↓all the↓ ↓ablest↓ Mathematicians <illegible word> have perused <illegible words> leasure & <illegible word> ↓who have had leasure to↓ examine the Book~~ Mathematical Principles of Philosophy ↓to the satisfaction of my readers: And if Nature be most simple & fully

new forms. The above corollary was slightly changed in the second edition of the *Principia* (Newton (1999). p. 809).

22 The corresponding manuscript material is to be found on CUL Add. Ms. 3970. 335^r-340^v.

> consonant to her self she observes the same method in regulating the motions of smaller bodies w^{ch} she doth in regulating those of the greater. ~~But what~~ This principle of nature being very remote from the <~~illegible words~~> Philosophers I forebore to describe it in ~~my Principles~~ ↓that said book↓, leas{t I s}hould be accounted an extravagant freak & so prejudice my readers against all th~~e~~↓o↓se things w^{ch} were y^{e} main designe of ~~my that~~ the Book: but now ~~those things being received by Mathematicians~~ & yet I hinted {?} both in the Preface ~~of~~ ↓& in↓ y^{e} book it self where I speak of the {?} of light & of y^{e} ~~rarefaction~~ <~~illegible words~~> elastic power of y^{e} Air: but {?} the design of y^{t} book being secured by the approbation of Mathematicians, {I have} not ~~doubted~~ ↓scrupled↓ to propose this Principle in plane words. The truth of this Hypothesis I assert not because I cannot prove it, but I think it very probable because a great part of the phænomena of nature ~~w^{ch}~~ do easily flow from it w^{ch} seem otherwise inexplicable (…). (CUL Add. Ms. 3970: 338^{r-v})

The above context sheds light on why Newton emphasized the transductive significance of Rule III when he introduced this *regula philosophandi* in the early 1690s: because it was crucial to his program of methodizing optics in a way comparable to his physico-mathematical treatment of rational mechanics.

Why then did Newton not address the transductive significance of Rule III in his published work? The answer seems to be that at another point in the early 1690s, which is difficult to date exactly, he came to question the methodological validity of transduction as a *generally* legitimate reasoning process from what is observable to what is unobservable. In a draft composed after CUL Add. Ms. 3965: 266^{r}, in which Rule III was referred to as "Axioma 4," Newton restricted Rule III to cover only generalizations taken from observable bodies: "Qualities of sensible bodies that cannot be intended and remitted are properties of all bodies."[23] On the verso side of the same manuscript, he clarified his motivation for doing so: "Some kind of insensible bodies, out of which sensible bodies could never be formed, can indeed be feigned to exist and for that reason their properties [i.e. the properties of insensible bodies] cannot be collected from the properties of sensible bodies" (CUL Add. Ms. 4005: 81^{v}).[24] As is widely known, Newton and Locke probably first met in 1689 they became close friends who exchanged theological and alchemical secrets and occasionally paid visits to one another (Westfall, 1980, 488-493). Although it is not possible to prove this directly, it might be the case that Newton changed his views on transduction by being exposed to John Locke's philosophy.[25] In a passage in the *Essay*, which could have drawn Newton's close attention, Locke pointed out:

23 "Qualitates ~~corporum omnium~~ ↓corporum sensibilium↓ quæ intendi et remitti neque sunt, ~~quatenus experiri licet, corporibus omnibus competunt~~ ↓sunt proprietates corporum omnium↓." (CUL Add. Ms. 4005. f. 81^{r} [early 1690s]).

24 My translation of: "ffingi quidem potest corporum ↓insensibilium↓ genus aliquod <~~illegible words~~> ↓existere↓ ex quibus corpora sensibilia nunquam formentur et ~~consequen~~{?} propterea horum proprietates ex corporum sensibilium proprietatibus colligi non potest."

25 Newton owned Locke's *Essay* (1689) (Harrison (1978). p. 181, item n° 967).

> If a great, nay far the greatest part of the several ranks of *Bodies* in the Universe, scape our notice by their remoteness, there are others that are no les concealed from us by their *Minuteness*. These insensible Corpuscles, being the active parts of Matter, and the great Instruments of Nature, on which depend not only all their secondary Qualities, but also most of their natural Operations, our want of precise distinct *Ideas* of their primary Qualities, keeps us in an uncurable Ignorance of what we desire to know about them. […] But whilst we are destitute of Senses acute enough, to discover the minute Particles of Bodies, and to give us *Ideas* of their mechanical Affections, we must be content to be ignorant of their properties and ways of Operation; nor can we be assured about them any farther, than some few Trials we make, are able to reach. (Locke (1975): 553-554)[26]

However this may be, one of the factors that contributed to Newton's increasing reservations concerning the unqualified use of transduction was that, once – after having finished the first edition of the *Principia* – he returned to his optical work, which would ultimately result in the publication of the *Opticks* in 1704, he came to realize the methodological differences between his use of transduction in Proposition VII, Book III of the *Principia,* which was based on Propositions LXXI-LXXVI, Book I, which deal with the attractive forces of spherical bodies (Newton, 1999: 590-597, 810-811),[27] and his (intended) use of transduction in his optical work.[28] Propositions LXXI-LXXVI basically enabled Newton to argue that the overall inverse-square centripetal force of a spherical body results from the individual inverse-square centripetal forces of each of the "parts [partes]" composing that sphere, and vice versa (Newton, 1999: 810). The transductive inferences enabled by these propositions are ultimately licensed by the quantity of matter's being an additive quality to which gravity is proportional. In the *Principia* Newton in other words constrained the use of transductive inferences to those transductive inferences that are based on well-defined physico-mathematical decompositions. Much later, in the corrections and additions to the second edition of the *Principia*, Newton worked on a list of new definitions which were to be inserted at the beginning of Book III. Newton provided definitions of the terms 'phenomenon', 'hypothesis', 'rule', 'body' and 'vacuum' (CUL Add. Ms. 3965: 420^r-422^v, 428^r, 430^r, 437^v, and 504^r). In one of the drafts in which he defined 'body', he called attention to the conditions under which the argument from induction, i.e. Rule III, can rightfully be applied:

26 I am indebted to George E. Smith for drawing my attention to this passage.

27 For discussion see Ducheyne (2012). pp. 97-103, pp. 148-149.

28 Newton also ran into problems with transduction in Book II of the Principia, as George E. Smith has documented: the microscopic models for the resistance force on a body arising from the fluid's inertia, which Newton introduced in Section VII, Book II of the Principia, lack rigorous justification (Smith (2004)). On the failure of transduction in Newton's optical work, see Shapiro (1993). esp. p. 45, p. 125, p. 134 and Ducheyne (2012). pp. 206-213.

> The chosen argument of induction from experiments and observations of sensible [things], on which experimental philosophy is based, cannot be applied to hypothetical or metaphysical entities, which are not phenomena, unless by hypothesis and, on that account, what is said in this book about bodies by the force of induction does not at all consider entities of this sort. Here only sensible [bodies] and their parts are treated [and it is] for this reason that the argument of induction may have [its] place in them only. Other [bodies] that cannot be perceived, but that are nevertheless hypothetically called bodies by some, should be more adequately treated in hypothetical metaphysics and philosophy. (CUL Add. Ms. 3965: 422ʳ)[29]

Here Newton pointed out that the argument of induction may have its place only in sensible bodies and *their parts*. It seems that he was implying that transductive arguments are only valid when they are based on the sort of physico-mathematical decompositions which he developed in the propositions on the attractive forces of spherical bodies. In Newton's optical work such decompositions were not available, because similar decompositions would entail hypothetical statements about the constituents of light and colors.

There is also a second factor that contributed to the fact that Newton never discussed the transductive significance of Rule III in his published work: the methodological criticism that was launched by Leibniz and others on the concept of gravitation. Once Leibniz launched his criticism, Newton tried to immunize the *Principia* from further criticism and increasingly came to position his natural philosophy as "experimental philosophy." In this context, he also came to emphasize such terms as 'phenomena', 'rules of philosophizing', 'deduction from phenomena', and the 'argument from induction', as Alan E. Shapiro has aptly drawn our attention to (2004). Newton's endeavor to protect the *Principia* from methodological criticism was accompanied by a cleansing of all hypothetical (or seemingly hypothetical) elements as far as possible. In this context he came to distinguish more rigidly between hypotheses, rules and phenomena in Book III of the second edition of the *Principia*.

On the corrections at the end of one of his two copies of the first edition of the *Principia*, Newton formulated Rule III as follows: "Laws and properties of all bodies upon which experiments are allowed to be made are laws and properties of

29 My translation of: "Argumentum Inductionis ab experimentis et sensibilium observationibus desumptum, in quo Philosophia experimentalis fundatur, ad entia vel hypothetica vel metaphysica quæ phænomena non sunt, applicari non potest nisi per hypothesin, ideoque quæ de corporibus vi Inductionis in hoc libro dicuntur, ad ejusmodi entia nil spectant. De solis sensibilibus et eorum partibus hic agitur propterea quod argumentum Inductionis in ijs solis locum habeat. Reliqua quæ non sentituntur sed per hypothesin tamen a nonnulis corpora nominantur, in Metaphysica et Philosophia hypothetica rectius tractanda sunt." The complete transcription of this definition and a reproduction of it is to be found in Ducheyne (2012). pp. 223-224.

all bodies universally" (WL NQ.16.200: unnumbered end-page).[30] Note that in this formulation of Rule III Newton referred to laws and that the criterion "remitti et intendi nequeunt" is absent in his formulation. The latter especially might be seen as indication that this version of Rule III was composed before Newton's formulation of Rule III on CUL Add. Ms. 3965: 266r.[31] The left margin on page 402 of the same copy however contains the following insertion: "Qualities of bodies that cannot be intended and remitted and that belong to all bodies upon which experiments are allowed to be made are qualities of all bodies universally" (WL NQ.16.200: 402).[32] His other copy of the first edition of the *Principia* contains the following version of Rule III: "Qualities of bodies that cannot be intended and remitted and that belong to all bodies upon which experiments are allowed to be made are properties of bodies universally." He then corrected "are properties" into "are to be taken as qualities" (CUL Adv.b.39.1: interleaved page between pp. 402-403).[33] He also changed the maxim's status: "Hypothesis III" became "Rule III". In its corrected form, this variant agrees exactly to the formulation of Rule III in the second and third edition of the *Principia* (Newton, 1713: 357; (1726): 387).

Whereas the function of Rule III is straightforward, the meaning of the words "intendi & remitti nequeunt" is far from being so. Here Newton did not exactly make it easy on his readers. In the commentary to Rule III, Newton stated that gravity is a universal quality and that gravity "is diminished [diminuitur] as bodies recede from the earth" (Newton, 1999: 795-796). If "intendi & remitti nequeunt" means "cannot be increased and diminished," as I. Bernard Cohen and Anne Whitman suggested in their translation, then Newton's assertion that gravity can be diminished implies that gravity is not a universal quality, which is rather problematic.[34] In the

30 My translation of: "Hypoth III. Leges ↓et proprietates↓ corporum omnium in quibus experimenta instituere licet sunt leges ↓et proprietates↓ corporum universorum."

31 Cf. Cohen (1971). p. 25.

32 My translation of: "Hypoth III. ~~Proprietates~~ Qualitates corporum quæ intendi et remitti nequeunt; quæque corporibus omnibus competent in quibus experimenta instituere licet, sunt qualitates corporum universorum." The variant corresponds to the an addition in the margin in the copy of the first edition of the *Principia* which Newton sent to Locke, to wit: "Hypoth III. Qualitas [sic] corporū quæ intendi et remitti nequeunt, quæque corporibus in quibus experimenta instituere licet, sunt qualitates corporum universorū." (WL Adv.b.1.6. p. 402; Cohen (1971). p. 24).

33 My translation of: "~~Hypoth~~↓Reg↓. III <u>Qualitates corporum quæ intendi et remitti nequeunt, quæque corporibus omnibus competunt in quibus experimenta instituere licet, ~~sunt proprietates~~ ↓pro qualitatibus↓ corporum universorum habendæ sunt</u>." Cf. Cohen (1971). p. 26.

34 In their defense, Cohen and Whitman were not the only ones that struggled with the meaning of "intendi & remitti nequeunt". Finocchiario and McGuire both claimed that qualities that cannot be intended and remitted are essential qualities (Finocchiaro (1974). p. 70; McGuire (1995). pp. 252-256). Spencer statement that the "intendi & remitti nequeunt" criterion refers to "constant" qualities of all bodies that are observable through experiment (Spencer (2004). p. 762) is misleading since constancy entails 'unchangeableness'.

commentary to Definition VII, i.e. the definition of the accelerative quantity of centripetal force, Newton explained that just as magnetism gravity varies as bodies are nearer or further from the body that attracts them:

> One example is the potency of a lodestone, which, for a given lodestone is greater at a smaller distance and less at a greater distance. Another example is the force that produces gravity, which is *greater in valleys and less on the peaks of high mountains and still less* (as will be made clear below) *at greater distances from the body of the earth, but which is everywhere the same at equal distances,* because it equally accelerates all falling bodies (heavy or light, great or small), provided that the resistance of the air is removed. (Newton, 1999: 407 [italics added])

Moreover, in Corollary 5 to Proposition VI, Book III, Newton argued that the force of gravity is "of a different kind from the magnetic force [diversi est generis a vi magnetica]." One of the differences which he mentioned is that magnetism, in contrast to gravity, "in one and the same body can be intended and remitted [in uno & eodem corpore intendi potest & remitti]" (Newton, 1999: 810). In Query 29 of the second edition of the *Opticks* (1717), Newton recorded that "And as Magnetism may be intended & remitted, & and is found only in the Magnet & in Iron: So this Virtue of refracting the perpendicular Rays is greater in Island Crystal, less in Crystal of the Rock & is not yet found in other Bodies" (Newton, 1717: 348).[35] Newton's published work informs us that he thought that magnetism can be intended and remitted and that gravity cannot be intended and remitted. What Newton's published work does not provide us with is an clear characterization of what is means for a quality (or a force) to be (or not to be) intended and remitted. A conceptual analysis of Newton's commentary to Rule III, as has once been suggested by Maurice A. Finocchiaro, is to no avail (1974: 66-73). We have to turn to unpublished sources to establish the meaning of "intendi & remitti nequeunt." In a memorandum composed on 5-7 May 1694, David Gregory reported on Newton's views on magnetism, as follows:

> Magnetic virtue *is destroyed* [interrumpitur] by a flame, and by heat: a rod of iron, either by standing long in a perpendicular position, or by cooling in an erect position, *acquires* [acquivit] magnetic virtue from the Earth. But it gets magnetic virtue too with a strong blow of a hammer at either extremity. If it is struck hard at one or other end the poles of the iron rod are interchanged: if it is struck in the middle (say with hammering at an anvil) it quite *loses* [amittit] its magnetism. (Turnbull, Scott, Hall, & Tilling, 1959-1977, III: 335/ 338)

The verbs which Gregory used are highly significant for they give us insight in what "intendi & remitti nequeunt" might mean. Gregory's memorandum seems to suggest that for Newton a quality that can be intended is a quality that can be "acquired" at a certain point in time and that a quality that can be remitted is a

35 The corresponding manuscript material is to be found on CUL Add. Ms. 3970. f. 272r.

quality that can be "destroyed" or "lost" at another point in time. This interpretation can be strengthened by consulting Newton's own words. Whilst Newton was composing the queries for the second edition of the *Opticks*, which was published in 1717, he wrote:

> All ↑sensible↑ bodies here below are heavy towards y^{e} Earth in proportion to the quantity of matter in ↓each of↓ them. Their gravity ↓in proportion to their matter↓ is not intended or remitted ↓in the same region of the earth by any variety of ↓↓fform↓↓ & therefore *it cannot be taken away* ↓~~I speak of bodies equally distant from y^{e} center of the earth~~↓ (CUL Add. Ms. 3970: 243^{v} [italics added])[36]

Note that Newton struck out this entire passage. On the assumption that we have just unearthed the meaning of "intendi & remitti nequeunt," we may easily understand why he did so: because upon rereading it he must have realized that the gravity of bodies cannot be intended and remitted *irrespective* of their distances from one another.

Whereas magnetism can be increased and diminished and can be acquired and taken away, gravity can be increased and diminished but cannot be acquired and taken away. Or put differently, there is no body that has a measurement of its gravitational force that is equal to zero. In an addition to the commentary to Rule III, which was added in the third edition of the *Principia*, Newton also introduced a third sort of force: forces that cannot be increased and diminished nor be acquired and taken away, namely the force of inertia, which is proportional to the quantity of matter of a body (Newton, 1999: 404):[37] "Yet I am by no means affirming that gravity is essential to bodies. By inherent force I mean only the force of inertia [vim insitam]. This is immutable [immutabilis]. Gravity is diminished [diminuitur] as bodies recede from the earth" (Newton, 1999: 796).[38] Newton distinguished between three sorts of forces: the magnetic force, which is a force that does not pertain to all bodies universally *and* that can be acquired *and* taken away in magnetic bodies, the force of gravity, which pertains to all bodies universally and which may vary, i.e. a force that cannot be acquired and taken away but can be increased and

36 In one of his copies of the first edition of the *Principia,* Newton wrote: "~~Gravitas ↓in Terram↓ est qualitas corporum omnium ↓quæ circa Terram sunt &↓ in quibus experimenta instituere licet & quantitati materiæ in singulis proportionalis existens non potest intendi et remitti & propterea per Hypoth III proprietas corporum universorum~~." (CUL Adv. B.39.1. interleaved page between pp. 402-403).

37 Cf. WL NQ.16.196. p. 358.

38 See CUL Add. Ms. 3965. f. 504^{r} and f. 519^{r} for the corresponding manuscript material, which is identical to the published version. The interleaved page between CUL Adv. B.39.1. pp. 402–3 contains the commentary to Rule III. It is nearly identical to the commentary as published in the second edition of the Principia.

diminished, and the force of inertia, which is a universal and immutable force, i.e. a force that cannot be acquired and taken away nor be increased and diminished. If my interpretation is correct, the late-medieval doctrine of latitudes of forms does little work in explaining the meaning of "intendi et remitti nequeunt" in Rule III.

Rule IV

In the third edition of the *Principia*, Rule IV made its appearance. It states:

> *In experimental philosophy, propositions gathered from phenomena by induction should be considered either exactly or very nearly true notwithstanding any contrary hypotheses, until yet other phenomena make such propositions either more exact or liable to exceptions*. (Newton, 1999: 796)[39]

Newton's comment reads: "This rule should be followed so that arguments based on induction may not be nullified [tollatur] by hypotheses" (Newton, 1999: 796). The goal of Rule IV is in part to protect propositions which are deduced from phenomena and rendered general by Rule III against hypothetical propositions, i.e. propositions that have not been deduced from phenomena. Instead, Rule IV asserts that propositions deduced from phenomena and rendered general by induction should be considered (provisionally) as exactly or as most closely as possibly true. Rule IV reminds us that Newton was approaching phenomena by a sequence of approximations, which is a typical feature of the *Principia*-style methodology.[40] In a passage in a never to be published fifth rule, Newton elaborated on the meaning of Rule IV, as follows:

> For if arguments from hypotheses would be admitted against inductions, inductive arguments, on which the whole of experimental philosophy is based, could always be overturned by contrary hypotheses. If a certain proposition collected by induction should be not sufficiently accurate, it ought be corrected, not by hypotheses but by phenomena of nature that are to be more widely and accurately observed. (CUL Add. Ms. 3965: 419^{v})[41]

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39 Translation of: "*In philosophia experimentali, propositiones ex phænomenis per inductionem collectæ, non obstantibus contrariis hypothesibus, pro veris aut accurate aut quamproxime haberi debent, donec alia occurrerint phænomena, per quæ aut accuratiores reddantur aut exceptionibus obnoxiæ*." (Koyré, Cohen & Whitman (1972), II. p. 555). This very formulation of Rule IV is to be found amongst Newton's corrections and additions to the second edition of the *Principia* (CUL Add. Ms. 3965. f. 504^{r} and f. 619^{r}).

40 See Cohen (1982), Smith (2002), Harper (2012). pp. 45-47, and Ducheyne (2012). Chapters 2 and 3.

41 My translation of: "Nam si argumenta ab Hypothesibus ↓contra Inductiones↓ admitterentur, argumenta ab Inductione↓um↓ in quibus tota Philosophia experimentalis fundatur ~~nihil valerent, sed~~ ↓~~Nam~~↓ per Hypotheses contrarias semper everti possent. Si Propositio~~nes~~ ↓aliqua↓ per Inductionem collectæ↓a↓ nondum s~~unt~~↓it↓ satis accuratæ↓a↓, corrigi debent, non per hypotheses, sed per phænomena naturæ fusius & accuratius observat↓nd↓æa." This passage is a commentary to the following rule: "In Philosophia experimentali contra Propositiones ex Phænomenis per Inductionem collectas non ~~est~~ sunt disputandum

Amongst Newton's additions and corrections to the second edition of the *Principia* there are four precursors of Rule IV, which are difficult to date exactly.[42] In one of these variants Newton pointed out that although, in stark contrast to geometrical demonstrations, inductive arguments are not necessarily "universal," they are stronger than hypotheses. If no exceptions occur to inductive generalizations, they are to be enunciated as holding generally [generaliter ennunciandæ sunt] (CUL Add. Ms. 3965: 428^{r}). In a passage in Query 31, which was added in the second edition of the *Opticks* (1717), Newton conveyed exactly the same point:

> And although the arguing from Experiments and Observations by Induction be no Demonstration of general Conclusions; yet it is the best way of arguing which the Nature of Things admits of, and may be looked upon as so much stronger, by how much the Induction is more general. And if no Exception occur from Phænomena, the Conclusion may be pronounced generally. But if at any time afterwards any Exception shall occur from Experiments, it may then begin to be pronounced with such Exceptions as occur. (Newton, 1979: 404; 1717: 380)[43]

ab Hypothesibus." CUL Add. Ms. 3965. f. 419^{r} contains a different fifth rule which stipulates the conditions under which statements are to be taken as hypotheses, namely: "Pro hypothesibus habenda sunt quæcunque ex rebus ipsis vel per sensus externos, vel per sensationem ~~mentis~~ cogitationum internarum non derivantur."

42 There is one precursor on CUL Add. Ms. 3965. 419^{r}, which reads: "In Philosophia experimentali, [~~Hypotheses contra argumentum Inductionis non sunt audiendæ,] sed]~~ Propositiones ~~per I~~ ex Phænomenis per Inductionem collectæ non obstantibus Hypothesibus, ~~contrarijs,~~ pro veris aut accuratè aut quamproxime haberi debent, donec alia occurrerint Phænomena per quæ aut accuratiores reddantur aut exceptionibus obnoxiæ. Hoc fieri debet ne ~~In~~ argumentum Inductionis ~~per Hypotheses~~ tollatur per Hypotheses." On CUL Add. Ms. 3965. f. 419^{v} there are two further precursors. The first of them, which Newton crossed out in its entirety, reads: "↓In Philosophia experimentali↓ Propositiones ex Phænomenis per Inductionem collectæ pro veris aut accurate aut quamproxime haberi debent donec alia occurrerint. ~ Phænomena per ↓quæ↓ aut accuratiores reddantur aut exceptionibus obnoxiæ ~~Quæ nondum sunt satis accuratæ, hæ per hypotheses emendari non debent sed ad~~ <~~illegible word~~> ~~revocari per phænomena ↓naturæ↓ fusius et accuratius observanda~~ ↓Argumenta ex↓ Hypothesibus contra argumentum Inductionus ~~nil valent~~ desumenda non sunt." The second reads: "In Philosophia ~~naturali~~ experimentali, ↓~~Hypotheses~~ contra argumentum Inductionis Hypotheses non sunt audi↓endæ sed↓↓ Propositiones ex Phænomenis per Inductionem collectæ ↓~~non sunt per hypotheses corrigendæ, per hypotheses; sed~~↓ pro veris aut accurate aut quamproxime haberi debent, donec alia occurrerint Phænomena per quæ aut accuratiores reddantur, aut exceptionibus obnoxiæ. Hoc fieri debet ne ~~Ind~~ Argumentum Inductionis per Hypotheses tollatur." The fourth precursor occurs on CUL Add. Ms. 3965. f. 428^{r}. It reads: "In Philosophia experimentali ~~Objectiones~~ ↓Hypotheses↓ contra argumenta ab experimentis per Inductionem desumpta ~~auderi~~ ↓admitti↓ non debent, ne ↓scilicet↓ Philosophia experimentalis cum Hypothetica confundatur. ~~[Demonstrationes Geometricæ universales sunt.]~~ Argumenta per ↓ab↓ Inductionem ~~non [fortiora sunt quam Hypotheses~~ non sunt Demonstrationes. ffortiora tamen sunt quam Hypotheses; & pro generalibus haberi debent nisi quatenus exceptiones ab experimentis desumptæ <~~illegible word~~> occurrant. Ideoque ubi nullæ occurrunt ejusmodi ~~ob~~↓ex↓~~j~~↓c↓~~ec~~↓p↓tiones, generaliter ennunciandæ sunt." Manuscript material which is indirectly related to Rule IV is to be found on CUL Add. Ms. 3970. f. 242^{r}, f. 243^{v}, f. 253^{r} and f. 621^{v}.

43 Corresponding draft material is on CUL Add. Ms. 3970. f. 621^{v}.

This shows that Newton felt the need to introduce something akin to Rule IV at least nine years before the publication of the third edition of the *Principia*. Although other factors cannot be excluded, in my view Newton definitely felt this need at some point between 18 and 28 March 1712/3.[44] As the second edition of the *Principia* was almost ready for publication, its editor, Roger Cotes, questioned Newton's application of the third law of motion in Corollary 1 to Proposition V, Book III in a letter dated on 18 March 1712/3. More precisely, Cotes questioned whether Newton was entitled to conclude from the third law of motion, which states that "every attraction is mutual," that "Jupiter will gravitate toward all its satellites, Saturn toward its satellites, and the earth will gravitate toward the moon, and the sun toward all the primary planets" (Newton, 1999: 806). He wrote:

> Suppose two Globes *A* & *B* placed at a distance from each other upon a Table, & that whilst *A* remains at rest *B* is moved towards it by an invisible Hand. A by-stander who observes this motion but not the cause of it, will say that *B* does certainly tend to the centre of *A*, & thereupon he may call the force the invisible Hand the Centripetal force of *B*, or the Attraction of *A* since ye effect appears the same as if it did truly proceed from a proper & real Attraction of *A*. But then I think he cannot by virtue of the Axiom [Attractio omnis mutua est] conclude contrary to his Sense and Observation, that the Globe *A* does also move towards Globe *B* & will meet it at the common centre of Gravity of both bodies. (Turnbull, Scott, Hall, & Tilling, 1959-1977, V: 392)

Cotes, in other words, pointed out that all that can be legitimately inferred from the third law is that, if a body is attracted towards a second body, the second body is counteracted by an equal and oppositely directed force, but not that the second body is attracted *by the first body* by an equal and oppositely directed reaction force (Stein, 1991: 217). In the draft version of the letter which Newton sent to Cotes on 28 March 1712/3, Newton responded as follows:

> But to admitt of such Hypotheses in opposition to rational Propositions founded upon Phænomena by Induction is to destroy all arguments taken from Phænomena by Induction & all Principles founded upon such arguments. And therefore as I regard not Hypotheses in explaining the Phenomena of nature so I regard them not in opposition to arguments founded upon Phænomena by Induction or to Principles setled upun such arguments. In arguing for any Principle or Proposition from Phænomena by Induction, Hypotheses are not to be considered. The Argument holds good till some Phænomena can be produced against it. This Argument holds good by the third Rule of philosophizing. And if we break that Rule, we cannot affirm any one general law of nature: we cannot so much as affirm that all matter is impenetrable. Experimental Philosophy reduces Phænomena to general Rules & looks upon the Rules to be general when they hold generally in Phænomena. It

44 Hereby I do not want to suggest that this was the first time that Newton considered the crux of the objection that Cotes raised. In fact, in talk presented at the international conference 'A great variety of admirable discoverys': *Newton's Principia* in the Age of Enlightenment George E. Smith has provided evidence that Newton anticipated this objection long before it became an issue in the correspondence between him and Cotes.

> is not enough to object that a contrary phænomenon may happen but to make a legitimate objection, a contrary phenomenon must be actually produced. Hypothetical Philosophy consists in imaginary explications of things & imaginary arguments for or against such explications, or against arguments of Experimental Philosophers founded upon Induction. (…) Experimental philosophy argues only from phænomena, draws general conclusions from the consent of phænomena, & looks upon the conclusion as general when ye consent is general without exception, tho the generality cannot be demonstrated a priori. (…) So in experimental Philosophy its proper to distinguish Propositions into Principles, Propositions & Hypotheses, calling those Propositions wch are deduced from Phænomena by proper Arguments & made general by Induction (the best way of arguing in Philosophy for a general Proposition) & those Hypotheses wch are not deduced from Phænomena by proper arguments. (Turnbull, Scott, Hall, & Tilling, 1959-1977, V: 398-399)

Newton was reminded by Cotes' intervention that Rule III needed to be supplemented by an account of what needs to be done if further natural-philosophical research shows that exceptions occur (or might occur) to a proposition deduced from phenomena and rendered general by induction (Cohen, 1971: 260).

In conclusion

As I have shown in my monograph, Newton's methodology was far from being static: it developed alongside with the new and never-ending challenges that he faced during his natural-philosophical career (cf. Ducheyne, 2012: Chapter 5). The additional *regulae* and their reformulations which Newton introduced in the second and third edition of the *Principia* bear testimony to some significant changes in his methodological thought. We have seen that in close agreement to the epistemic and provisional undertone of Rule III, which in the second edition of the *Principia* stipulated the conditions under which certain qualities "*are to be taken*" as universal qualities, Newton changed the formulation of Rule II from "causes of natural effects of the same kind *are* the same" into "the causes *assigned to* natural effects of the same kind *must be, so far as possible*, the same" in the third edition of the *Principia*. We have also seen from the editorial history which has been provided that in the early 1690s Newton considered Rule III as an inductive rule with clear transductive significance. Shortly thereafter, he came to question the methodological validity of transduction as a *generally* legitimate reasoning process and for this reason he did not point to the transductive significance of Rule III in the published versions of his work. Rule IV, which was introduced in the third edition of the *Principia,* dictates how legitimately established inductive generalizations are to be treated in view of novel natural-philosophical research: they should either be taken as exactly or as closely as possibly true so that, as George E. Smith has amply highlighted (Smith, 2002: 159-160), deviations from the proportions which they stipulate can be taken into account in the establishment of a more fine-grained physico-mathematical model or, when legitimate exceptions occur from phenomena, their inductive range should be de-generalized. Rule IV challenged natural philosophers to render legitimately established inductions more exactly or to de-generalize them if required.

Feynman is correct that Newton's argument for the theory of universal gravitation was "probably one of the most far-reaching generalizations of the human mind." At the same time, it was also one of the most thought-through generalizations in the history of science, which can be gathered from the careful attention which Newton dedicated to the rules that were required to justify that generalization.

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