Active vibration control of vehicle suspension systems using sliding modes, differential flatness and generalized proportional-integral control

Control activo de vibraciones de sistemas de suspensión de vehículos usando modos deslizantes, planitud diferencial y control proporcional-integral generalizado

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Abstract

This paper presents a design approach of robust active vibration control schemes for vehicle suspension systems using differential flatness, sliding modes and Generalized Proportional-Integral control techniques to attenuate undesirable vibrations induced by irregular road disturbances. Two control schemes are proposed: one employing position and velocity measurements and other requiring only position measurements. Integral reconstruction of the time derivatives of the flat output up to third order is proposed to avoid the use of velocity and acceleration sensors, as well as information of unknown road disturbances.

----- *Keywords:* Active vehicle suspension system, sliding modes, differential flatness, generalized proportional-integral control

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Resumen

En este artículo se presenta un enfoque de diseño de esquemas de control activo robusto para sistemas de suspensión de vehículos, usando técnicas de control Proporcional-Integral Generalizado, modos deslizantes y planitud diferencial, para atenuar vibraciones indeseables inducidas por perturbaciones de terreno irregular. Se proponen dos esquemas de control: uno de ellos emplea mediciones de posición y de velocidad y el otro requiere únicamente mediciones de posición. Se propone la reconstrucción integral de las derivadas de la salida plana para evitar el uso de sensores de velocidad y de aceleración, así como la información de perturbaciones de terreno desconocidas.

----- *Palabras clave:* sistema de suspensión activa de vehículo, modos deslizantes, planitud diferencial, control proporcional-integral generalizado

Introduction

The main objective on the active vibration control problem of vehicle suspension systems is to get security and comfort for the passengers by reducing to zero the vertical acceleration of the body of the vehicle. An actuator incorporated to the suspension system applies control forces to the vehicle body for reducing its vertical acceleration in active or semi-active way. The topic of active vehicle suspension control system has been quite challenging over the years and we refer the reader to some of the fundamental works in the vibration control areaand the references therein [1]. Some research works in this field propose control strategies like LQR in combination with nonlinear backstepping control techniques [2], which require information of the state vector (vertical positions and speeds of the tire and car body). A reduced order controller is proposed in [3] to decrease the implementation costs without sacrificing the security and the comfort by using accelerometers for measurements of the vertical movement of the tire and car body. In [4], a controller of variable gain that considers the nonlinear dynamics of the suspension system is proposed. It requires measurements of the vertical position of the car body and the tire, and the estimation of other states and the profile of the road. In addition, some interesting semiactive vibration control schemes, based on Electro-Rheological (ER) and Magneto Rheological (MR) dampers, have been proposed and implemented on commercial vehicles in [5, 6].

On the other hand, many dynamical systems exhibit a structural property called differential flatness. This property is equivalent to the existence of a set of independent outputs, called flat outputs and equal in number to the control inputs, which completely parameterizes every state variable and control input [7]. By means of differential flatness the analysis and design of controller is greatly simplified. In particular, the combination of differential flatness with sliding modes, which is extensively used when a robust control scheme is required, e.g., parameter uncertainty, exogenous disturbances and un-modeled dynamics [8], qualifies as an adequate robust control design approach to get high vibration attenuation level in active vehicle suspension systems. In this paper is proposed asuitable combination of differential flatness and sliding modes with the Generalized Proportional-Integral (GPI) Control approach to design efficient and robust active vibration control schemes for vehicle suspension systems that only require information of the flat output.

GPI control, or control based on integral reconstructors of the unmeasured observable state variables, for the regulation and trajectory tracking tasks on time invariant linear systems was introduced by Fliess and co-workers in [9]

as a generalization of classical PID control. This control design methodology has been extended to time-varying linear systems in [10] and, also, to a certain class of nonlinear systems [11]. The main objective is to avoid the use of traditional asymptotic state observers, or digital computations based on output samplings, therefore, the integral reconstructor-based feedback control laws are purely analog, requiring only measurements of the system output. The integral reconstruction of the state variables is carried out by means of elementary algebraic manipulations of the system model along with suitable invocation of the observability property. The purpose of the integral reconstructors is to get expressions for the unmeasured states in terms of inputs, outputs, and sums of a finite number of iterated integrals of the measured variables. In essence, constant errors and iterated integrals of such constant errors are allowed on these reconstructors. The current states thus differ from the integrally reconstructed states in time polynomial functions of finite order, with unknown coefficients related to the neglected and unknown, initial conditions. The use of these integral reconstructors in the synthesis of a state feedback controller needs suitable counteracting the effects of the implicit time polynomial errors. The destabilizing effects of the state estimation errors can be compensated by additively complementing a purely state feedback controller with a linear combination of a sufficient number of iterated integrals of the output tracking error, or output stabilization error. The closed loop stability is guaranteed by a simple characteristic polynomial assignment to the higher order compensated controllable and observable input-output dynamics.

In this paper is presented a design approach of robust active vibration control schemes for vehicle suspension systems using differential flatness, sliding modes and Generalized Proportional-Integral control techniques to attenuate undesirable vibrations induced by unknown irregular road disturbances. In the control problem formulation is assumed that these perturbations are not available to implement a control scheme. Two control schemes are proposed. The first one requires information of the time derivatives of the flat output up to third order or measurements of all the state variables (displacements and velocities). The second controller only requires information of the flat output; the main idea is the use of integral reconstruction of the non-measurable state variables or the time derivatives of the flat output instead of state observers. This design approach results in robust active vibration control schemes against parameter uncertainties and exogenous disturbances. Simulation results are included to show the dynamic performance and robustness of the active control schemes proposed for a quarter-vehicle active suspension system.
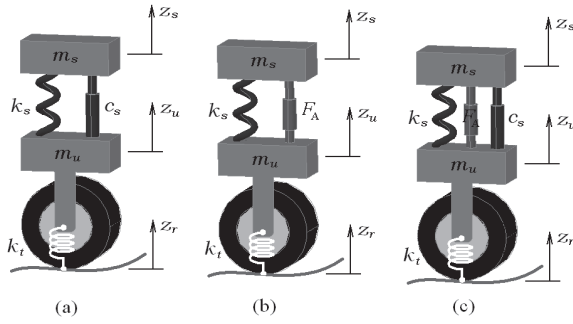
Quarter-Car suspension systems

Mathematical model of passive suspension system

A schematic diagram of a two degrees-of-freedom quarter-vehicle suspension system is shown in figure 1(a). The mathematical model of the passive suspension system is described by

$$m_s \ddot{z}_s + c_s (\dot{z}_s - \dot{z}_u) + k_s (z_s - z_u) = 0 \qquad (1)$$

$$m_u \ddot{z}_u - c_s (\dot{z}_s - \dot{z}_u) - k_s (z_s - z_u) + k_t (z_u - z_r) = 0 \quad (2)$$

where the sprung mass m_s represents the mass of the car-body part, the unsprung mass m_u denotes the mass of the assembly of the axle and wheel, c_s is the damper coefficient of suspension, k_s and k_t are the spring coefficients of suspension and the tire, respectively, z_s is the displacement of the sprung mass, z_u is the displacement of the unsprung mass and z_r is the terrain input disturbance.

Figure 1 Quarter-car suspension systems: (a) Passive Suspension System, (b) Active Electromagnetic Suspension System and (c) Active Hydraulic Suspension System

Mathematical model of active electromagnetic suspension system

A schematic diagram of a quarter-car active electromagnetic suspension system is illustrated in Figure 1(b). The electromagnetic actuator replaces the damper, forming a suspension with the spring [12]. The friction force of an electromagnetic actuator is neglected. The mathematical model of an electromagnetic active suspension system is given by

$$m_s \ddot{z}_s + k_s (z_s - z_u) = F_A \qquad (3)$$

$$m_u \ddot{z}_u - k_s (z_s - z_u) + k_t (z_u - z_r) = -F_A \qquad (4)$$

where F_A is the electromagnetic actuator force, which is considered as the control input.

Mathematical model of hydraulic active suspension system

Figure 1(c) shows a schematic diagram of a quarter-car active hydraulic suspension system. The mathematical model of this active suspension system is given by

$$m_s \ddot{z}_s + c_s (\dot{z}_s - \dot{z}_u) + k_s (z_s - z_u) = -F_f + F_A \qquad (5)$$

$$m_u \ddot{z}_u - c_s (\dot{z}_s - \dot{z}_u) - k_s (z_s - z_u) + k_t (z_u - z_r) = F_f - F_A \qquad (6)$$

where F_A is the hydraulic actuator force and F_f is the friction force generated by the seals of the piston with the cylinder wall inside the actuator. This friction force has a significant magnitude ($> 200N$) and cannot be ignored [5, 12].

Defining the state variables $x_1 = z_s$, $x_2 = \dot{z}_s$, $x_3 = z_u$ and $x_4 = \dot{z}_u$, one obtainsthe following state-space description:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\dfrac{k_s}{m_s} & -\dfrac{c_s}{m_s} & \dfrac{k_s}{m_s} & \dfrac{c_s}{m_s} \\ 0 & 0 & 0 & 1 \\ \dfrac{k_s}{m_u} & \dfrac{c_s}{m_u} & -\dfrac{k_s+k_t}{m_u} & -\dfrac{c_s}{m_u} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{1}{m_s} \\ 0 \\ -\dfrac{1}{m_u} \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dfrac{k_t}{m_u} \end{bmatrix} z_r \quad (7)$$

The net force provided by the hydraulic actuator as control input $u = F_A - F_f$ is the difference between the hydraulic force F_A and the friction force F_f.

Sliding mode-differential flatness control of hydraulic suspension system

The hydraulic suspension system is controllable and, there fore, differentially flat [13] with a flat output constructed as a linear combination of the displacements of the body of the car and wheel [14, 15]:

$$F = m_s x_1 + m_u x_3 \qquad (8)$$

In our analysis z_r is considered as an unknown exogenous perturbation signal due to irregular road surfaces with uniformly bounded magnitude for all $t \gg 0$, i.e., $\max_{t \geq 0} |z_r(t)| \leq \lambda$, which could be suppressed or at least attenuated by the sliding mode-based GPI control action. In fact, our design approach pursues to exploit the sliding mode control methodology for design of robust suspension system controllers with respect to unmodeled additive bounded perturbation input signals and parametric uncertainties.

Then, all the unperturbed system variables can be parameterized in terms of the flat output and a finite number of its time derivatives as follow,

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$$x_1 = \frac{1}{m_s}\left(F + \frac{m_u}{k_t}\ddot{F}\right), \quad x_2 = \frac{1}{m_s}\left(\dot{F} + (\frac{m_u}{k_t})F^{(3)}\right), \quad x_3 = -\frac{1}{k_t}\ddot{F}, \quad x_4 = -\frac{1}{k_t}F^{(3)}$$

$$u = \frac{m_u}{k_t}F^{(4)} + \left(\frac{c_s m_u}{k_t m_s} + \frac{c_s}{k_t}\right)F^{(3)} + \left(\frac{k_s m_u}{k_t m_s} + \frac{k_s}{k_t} + 1\right)\ddot{F} + \frac{c_s}{m_s}\dot{F} + \frac{k_s}{m_s}F$$

From this differential parameterization, we get the following input-output differential equation:

$$F^{(4)} = \frac{1}{\eta_1}u - \frac{\eta_2}{\eta_1}F^{(3)} - \frac{\eta_3}{\eta_1}\ddot{F} - \frac{\eta_4}{\eta_1}\dot{F} - \frac{\eta_5}{\eta_1}F \quad (9)$$

where

$$\eta_1 = \frac{m_u}{k_t}, \quad \eta_2 = \frac{c_s m_u}{k_t m_s} + \frac{c_s}{k_t}, \quad \eta_3 = \frac{k_s m_u}{k_t m_s} + \frac{k_s}{k_t} + 1, \quad \eta_4 = \frac{c_s}{m_s}, \quad \eta_5 = \frac{k_s}{m_s}$$

Now, consider the traditional sliding surface,

$$\sigma = F^{(3)} + \beta_2\ddot{F} + \beta_1\dot{F} + \beta_0 F \quad (10)$$

which is chosen so that the error dynamics restricted to $\sigma = 0$ is governed by the desired linear differential equation,

$$F^{(3)} + \beta_2\ddot{F} + \beta_1\dot{F} + \beta_0 F = 0 \quad (11)$$

The design gains $\beta_2,\ldots,\beta_0$ are selected to verify that the associated characteristic polynomial $s^3 + \beta_2 s^2 + \beta_1 s + \beta_0$ be Hurwitz. As a consequence, the error dynamics on the switching surface $\sigma = 0$ is globally asymptotically stable. The sliding surface $\sigma = 0$ is made globally attractive with the continuous approximation to the discontinuous sliding mode controller as given in [16], i.e., by forcing the system to satisfy the dynamics,

$$\dot{\sigma} = -\mu[\sigma + \gamma sign(\sigma)] \quad (12)$$

where μ, γ denote positive real constants and "sign" is the standard *signum* function. Indeed, for $\sigma \neq 0$, and using the Lyapunov function $V = \frac{1}{2}\sigma^2$,

we have that $\dot{V} = \sigma\dot{\sigma} = -\mu\sigma^2 - \mu\gamma|\sigma| < 0$.

Therefore a sliding regime is guaranteed to exist on $\sigma = 0$ for all time t after the finite hitting time t_h [8]. Moreover, if the differentially flat linear vehicle suspension system dynamics is subjected to the bounded unknown perturbation input signal ξ (t), it easy to prove, using the Lyapunov function $V = \frac{1}{2}\sigma^2$ and choosing

$W = \mu\gamma > \sup_t|\xi(t)|$, the asymptotic convergence of σ to the sliding surface σ=0. Indeed,

$$\dot{V} = \sigma\dot{\sigma} \leq -\mu\sigma^2 - \left(W - \sup_t|\xi(t)|\right)|\sigma| < 0.$$

One then obtains from (9) the following sliding-mode controller:

$$u = \eta_1 v + \eta_2 F^{(3)} + \eta_3\ddot{F} + \eta_4\dot{F} + \eta_5 F \quad (13)$$

with

$$v = -\beta_2 F^{(3)} - \beta_1\ddot{F} - \beta_0\dot{F} - \mu[\sigma + \gamma sign(\sigma)]$$

This controller requires measurements of all the state variables of the suspension system, z_s, $\dot{z}_s$, z_u and $\dot{z}_u$ corresponding to the vertical positions

and velocities of the body of the car and the tire, respectively.

In addition, the controller (13) can also be implemented for the active electromagnetic suspension system given by the equations (3) and (4), where the damper and the hydraulic actuator are replaced by an electromagnetic actuator, forming with the spring an oil-free suspension [12]. In this case, the control input u is the force F_A supplied by the electromagnetic actuator, i.e., $u = F_A$.

Sliding mode-GPI control of hydraulic suspension system

An integral input-output parameterization of the time derivatives of the flat output, is given, modulo initial conditions, by

$$\hat{F}^{(3)} = \frac{1}{\eta_1}\int u - \frac{\eta_2}{\eta_1}\hat{\hat{F}} - \frac{\eta_3}{\eta_1}\hat{F} - \frac{\eta_4}{\eta_1}F - \frac{\eta_5}{\eta_1}\int F$$

$$\hat{\hat{F}} = \frac{1}{\eta_1}\int^{(2)}u - \frac{\eta_2}{\eta_1}\hat{F} - \frac{\eta_3}{\eta_1}F - \frac{\eta_4}{\eta_1}\int F - \frac{\eta_5}{\eta_1}\int^{(2)}F$$

$$\hat{F} = \frac{1}{\eta_1}\int^{(3)}u - \frac{\eta_2}{\eta_1}F - \frac{\eta_3}{\eta_1}\int F - \frac{\eta_4}{\eta_1}\int^{(2)}F - \frac{\eta_5}{\eta_1}\int^{(3)}F$$

These expressions are obtained by successive integrations of the equation (9). For simplicity, we have denoted the integral $\int_0^t \phi(\tau)\,d\tau$ by $\int \phi$ and $\int_0^t\int_0^{\sigma_1}\cdots\int_0^{\sigma_{n-1}}\phi(\sigma_n)\,d\sigma_n\cdots d\sigma_1$ by $\int^{(n)}\phi$ with n a positive integer.

The relationships between the state variables and the integrally reconstructed state variables are given by

$$F^{(3)} = \hat{F}^{(3)} + F^{(3)}(0)t^2 + F^{(3)}(0)t + 2\ddot{F}(0)t + \ddot{F}(0) + 2\dot{F}(0)$$

$$\ddot{F} = \hat{\hat{F}} + \frac{1}{2}F^{(3)}(0)t^2 + F^{(3)}(0)t + \ddot{F}(0)t + \ddot{F}(0) + \dot{F}(0)$$

$$\dot{F} = \hat{F} + \frac{1}{2}F^{(3)}(0)t^2 + \ddot{F}(0)t + \dot{F}(0)$$

where $F^{(3)}(0)$, $\ddot{F}(0)$ and $\dot{F}(0)$ are all real constants depending on the unknown initial conditions.

The sliding surface inspired on the GPI control can be proposed as,

$$\hat{\sigma} = \hat{F}^{(3)} + \alpha_5\hat{\hat{F}} + \alpha_4\hat{F} + \alpha_3 F + \alpha_2\int F + \alpha_1\int^{(2)}F + \alpha_0\int^{(3)}F \tag{14}$$

The last integral term yields error compensation, eliminating destabilizing effects, those of the structural estimation errors. The ideal sliding condition $\hat{\sigma} = 0$ results in a sixth order dynamics,

$$F^{(6)} + \alpha_5 F^{(5)} + \alpha_4 F^{(4)} + \alpha_3 F^{(3)} + \alpha_2\ddot{F} + \alpha_1\dot{F} + \alpha_0 F = 0 \tag{15}$$

The gains of the controller $\alpha_5, \ldots, \alpha_0$ are selected so that the associated characteristic polynomial

$$s^6 + \alpha_5 s^5 + \alpha_4 s^4 + \alpha_3 s^3 + \alpha_2 s^2 + \alpha_1 s + \alpha_0 \quad \text{is}$$

Hurwitz. As a consequence, the error dynamics on the switching surface $\hat{\sigma} = 0$ is globally asymptotically stable.

The sliding surface $\hat{\sigma} = 0$ is made globally attractive with the continuous approximation to the discontinuous sliding mode controller as given in [16], i.e., by forcing to satisfy the dynamics,

$$\dot{\hat{\sigma}} = -\mu[\hat{\sigma} + \gamma sign(\hat{\sigma})] \tag{16}$$

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where μ and γ denote real positive constants. As before, the sliding surface is globally attractive,

$$\widehat{\sigma}\dot{\widehat{\sigma}} = -\mu\widehat{\sigma}^2 - \mu\gamma|\widehat{\sigma}| < 0 \quad \text{for} \quad \widehat{\sigma} \neq 0 \text{. Then the}$$

following sliding-mode controller is obtained:

$$u = \eta_1 v + \eta_2 \widehat{F}^{(3)} + \eta_3 \dot{\widehat{F}} + \eta_4 \widehat{F} + \eta_5 F \quad (17)$$

with

$$v = -\alpha_5 \widehat{F}^{(3)} - \alpha_4 \dot{\widehat{F}} - \alpha_3 \widehat{F} - \alpha_2 F - \alpha_1 \int F - \alpha_0 \int^{(2)} F - \mu[\widehat{\sigma} + \gamma sign(\widehat{\sigma})]$$

This controller requires only the measurement of the variables of the states z_s and z_u corresponding to the vertical displacements of the car body and the wheel, respectively.

Instrumentation of active suspension system

Measurements Required

The variables only required for implementation of the proposed Sliding Mode-GPI controller are the vertical displacement of the body of the car z_s, and the vertical displacement of the wheel z_u. These variables are needed to be measured by sensors. On the other hand, for implementation of the proposed Sliding Mode-Differential Flatness controller, are needed to measure the vertical displacement of the body of the car and the wheels, as well as the velocities of the body of the car $\dot{z}_s$, and the wheel $\dot{z}_u$.

Using Sensors

In [17], the use of sensors in experimental vehicle platforms, as well as in commercial vehicles is presented. The most common sensors, used for measuring the vertical displacement of the body of the car and the wheels, are laser sensors. This type of sensor could be used to measure the variables z_s and z_s needed for implementation of the controllers. Accelerometers or other types of sensors are needed to get information of the velocity signals $\dot{z}_s$ and $\dot{z}_u$. In the Sliding Mode-

GPI Control scheme, the velocity variables are estimated with the use of integral reconstructors from the knowledge of the control input, the flat output and the differentially flat system model.

A schematic diagram of the instrumentation of the active suspension system is illustrated in figure 2.
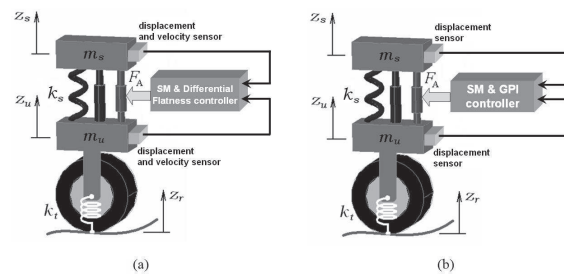
Figure 2 Schematic diagram of the instrumentation of the active suspension system: (a) Sliding Mode-Differential Flatness Control, (b) Sliding Mode-GPI Control

Simulation results

The simulation results were obtained by means of MATLAB/Simulink®, with the Runge-Kutta numerical method and a fixed integration step of 1 *ms*.

Parameters and type of road disturbance

The numerical values of the quarter-car suspension model parameters [18] chosen for the simulations are shown in table 1.

Table 1 Vehicle suspension system parameters for a quarter-car model

Parameter	Value
Sprung mass (m_s)	282 [kg]
Unsprung mass (m_u)	45 [kg]
Spring stiffness (k_s)	$17900 \left[\dfrac{N}{m}\right]$
Damping constant (c_s)	$1000 \left[\dfrac{N \cdot s}{m}\right]$
Tire stiffness (k_t)	$165790 \left[\dfrac{N}{m}\right]$

The following trajectory was utilized to simulate the unknown exogenous disturbance excitations due to irregular road surfaces [18]

$$z_r = a\frac{1-cos(8\pi t)}{2} \qquad (18)$$

where a is the bump amplitude, which is set to be $a = 0.11$ [m] for $0.5 \le t \le 0.75$, $a = 0.55$ [m] for $3.0 \le t \le 3.25$ and 0 otherwise.

The desired robust performance of the closed-loop vehicle suspension system is to reduce as much as possible the car-body vibrations induced by the road surface perturbation.

Sliding mode-differential flatness controller performance

It is desired to stabilize the system in the positions $z_s = 0$ and $z_u = 0$. The controller gains (13) were obtained by forcing the closed-loop characteristic polynomial to be given by the following *Hurwitz* polynomial:

$$p_{d1}(s) = (s+p)(s^2 + 2\zeta_1\omega_{n1}s + \omega_{n1}^2) \qquad (19)$$

with $p = 100$, $\zeta_1 = 0.5$, $\omega_{n1} = 90$, $\mu = 95$ and $\gamma = 95$. The simulation results are illustrated in figure 3. It can be seen a high vibration attenuation level of the active vehicle suspension system (car-body acceleration and position) compared with the passive counterpart. Moreover, one can observe the active suspension deflection response to compensate the road surface perturbation force, and that $\sup_t |x_1 - x_3| < a$, where a is the bump amplitude.
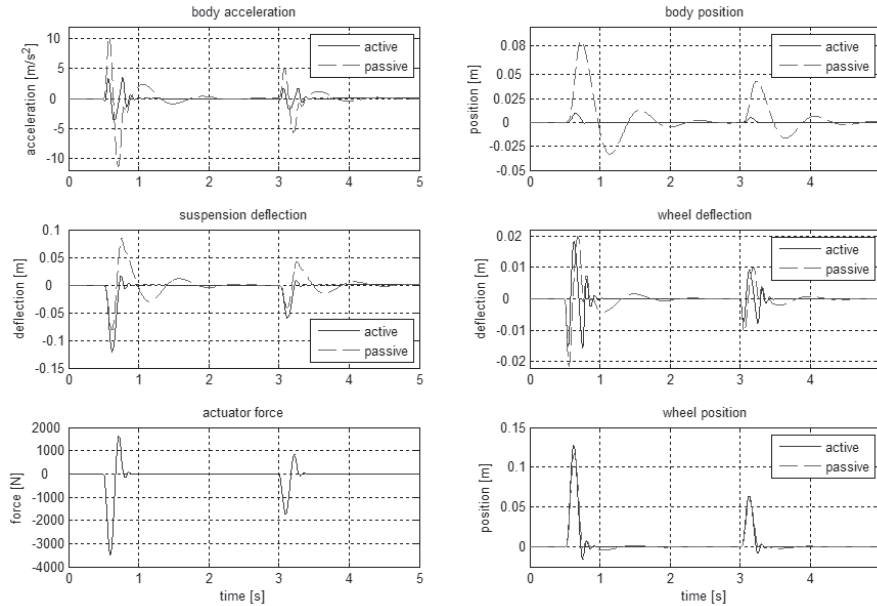
Figure 3 Simulation results of Sliding Modebased Differential Flatness controller of hydraulic suspension system

Sliding mode-GPI controller performance

It is desired to stabilize the system in the positions $z_s = 0$ and $z_u = 0$. The controller gains (17) were obtained by forcing the closed-loop characteristic polynomial to be given by the following *Hurwitz* polynomial:

$$p_{d2}(s) = (s + p_1)(s + p_2)(s^2 + 2\zeta_2\omega_{n2}s + \omega_{n2}^2)^2$$

with $p_1 = 90$, $p_2 = 90$, $\zeta_2 = 0.9$, $\omega_{n2} = 70$, $\mu = 95$ and $\gamma = 95$. The performance of the sliding mode based GPI controller is depicted in figure 4. One can see the high attenuation level of road-induced vibrationswith respect to passive suspension system.
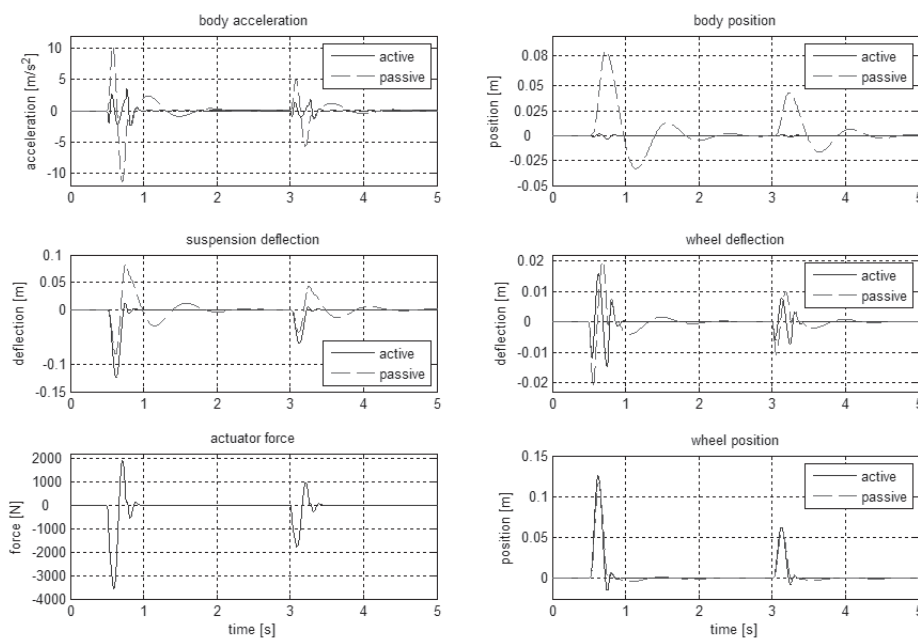
Figure 4 Simulation results of Sliding Mode based GPI controller of hydraulic suspension system

Conclusions

In this paper we have presented a design approach of robust active vibration control schemes for vehicle suspension systems based on Generalized Proportional-Integral control, differential flatness and sliding modes. Two control schemes have been proposed to attenuate the vibrations induced by unknown exogenous disturbance excitations due to irregular road surfaces. Integral reconstruction is employed to get structural estimates of the time derivatives of the flat output, needed for the implementation of the controllers proposed. The simulation results show that the stabilization of the vertical position of the quarter of car is obtained within a period of time much shorter than that of the passive suspension system. The fast stabilization with amplitude in acceleration and position of the body of the car is observed. Finally, the robustness of the controllers to stabilize to the system before the unknown disturbance is verified.

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