A novel graphical and analytical method for the kinematic analysis of fourth class Assur groups

Un método grafo-analítico para el análisis cinemático de los grupos de Assur de cuarta clase

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Abstract

A method for the kinematic analysis of a fourth class Assur group, using a combination of graphical and analytical methods, is presented in this paper. The solution is obtained through a method in which two special Assur points are used. A mechanism of 1 DOF with a fourth class group is considered as an example to develop the proposed method. The results of this method are in agreement with the results obtained by a dynamic simulation program. Since there are no solutions for fourth class structural groups in the literature, this method allows developing a complete modular procedure for the kinematic analysis of mechanisms, with the methodological advantages that this type of solution offers.

----- *Keywords:* Kinematic analysis, fourth class Assur group, structural analysis

Resumen

En este artículo se presenta un método para el análisis cinemático de un grupo de cuarta clase, utilizando un método grafo-analítico. La solución es obtenida utilizando dos puntos especiales de Assur. Se utiliza como ejemplo un mecanismo de 1 GDL con un grupo de Assur de cuarta clase. Los resultados obtenidos coinciden plenamente con los resultados obtenidos al utilizar un programa de simulación dinámica. Ya que este tipo de tareas para los grupos estructurales de cuarta clase, no se resuelve en la literatura, el método propuesto permite el desarrollo de un análisis modular completo para el análisis cinemático de mecanismos, con las ventajas metodológicas que ofrecen este tipo de soluciones.

----- *Palabras claves:* Análisis cinemático, grupo de Assur de cuarta clase, análisis estructural

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Introduction

A planar mechanism can be described with the help of different modules such as the frame, the driver links, and Assur groups of second, third, fourth class, etc. [1]. The essential classification of mechanisms was proposed by L V Assur in 1914. This scientist proposed and developed a method for the creation of mechanisms as a consecutive superposition of kinematic chains that have specific structural properties.

Mechanism assembly consists of connecting the group (all the joints that are external to the group) to a base mechanism. The joints are connected to some mobile links, which has a defined movement law, or to the structure of the mechanism. The kinematic analysis (positions, velocities, and accelerations) can be performed through the consecutive determination of each Assur group, when the movement laws of the driving links are given. This method is known as the modular method [2], in which the kinematic equations are formulated and solved separately for each module. The order of the solutions of the equations is defined by the structure of the mechanism. The analysis of positions for each Assur group consists of determining all the possible configurations, when the positions of the external joints and the lengths of the links are given. The solution for second, third and fourth class Assur groups are described in works such as [3-8].

The kinematic analyses for an Assur group consists of determining the angular velocities and accelerations of the links and the linear velocities and accelerations of the internal joints of the group. Kinematic analysis, using graphical methods, for second and third class Assur groups are described in several works [1, 8, 9]. Analytical methods for second and third class groups are described in [3, 9]. Some authors propose numerical methods for the kinematic and dynamic analysis of multibody systems [10] based on natural coordinates. The numerical methods have contributed to the development of dynamic simulation computer programs. Buskiewicz [11] proposes an algorithm that compiles structural and kinematic analysis; this is numerically easy to implement. The kinematic analysis is based on standard kinematic equations that are functions of velocities and accelerations, allowing a modular and kinematic analysis for the different Assur groups that constitute the mechanism.

Kinematic analyses of a fourth class Assur group are studied in several papers [9-12]. For this group is also possible to use analytical methods, formulate the kinematic equations of the group, and then derive them with respect to time [13]. Once the Jacobian matrix is obtained, velocities and accelerations for the group are easy to determine by solving a linear equation system.

From the literature review it is concluded that the grapho-analytical kinematic solutions for the fourth class Assur groups practically do not exist in the specialized literature, that fact makes difficult to develop a complete modular method for this kind of task.

A new method for the kinematic analyses of a fourth class Assur group, based on the combination of graphical and analytical methods, is proposed in this paper. Initially, a graphical procedure is used to determine the velocities and accelerations for points that are called *special Assur points*; there are two special points, one for each link with internal joints (mentioned in this paper as *closure links*) of the fourth class Assur group. A system of four linear equation is obtained from the kinematic diagrams (velocity or acceleration): two equations for each special point concerned with the relative velocities or the relative accelerations. The system of linear equation has four unknown variables that are the angular velocities or the angular accelerations of the group links. Once the angular velocities or accelerations for the links related to the group are obtained, the velocity or acceleration for any point that belongs to some of their links can be calculated. The proposed procedure is verified in this work by taking as examples one mechanism of a 1 DOF with a fourth class Assur group.

Methodology

The determination of velocities and accelerations for a fourth class Assur group can be obtained using the *Assur special point method*. A fourth class Assur group, shown in figure 1.a, consists of two ternary links, called here *drag members*, two joints, which are an internal and an external joint, and two binary links, both being internal joints. The external joint of the ternary links, as shown in figure 1.a, are joints with links 1 and 6 of the base mechanism. Given the velocities and accelerations of the external points of the group, joints B and E of figure 1, the kinematic analysis of a fourth class Assur group consists of determine the angular velocities and accelerations for the links of the group.

The first step of the analysis consists of determining the special Assur points for the binary links of the group with internal joints, links 3 and 5 (called in this paper *closure links*). The special Assur point of link 3 is obtained by extending line BC, which belongs to link 2, and ED, which belongs to link 4, to find the intersection point S_3; this point is considered to belong to link 3. A similar procedure is followed to obtain point S_5; this point is considered to belong to link 5. In this case, the point can be found by projecting line BG, which belongs to link 2, and line EF, which belongs to link 4.

Analysis of velocities

Now, it is possible to determine the velocities for points S_3 and S_5 using the velocity diagram shown in figure 1.b. Therefore, the segments *pb* and *pe* can be drawn from point *p*. These segments represent the given velocities of points B and E, at the chosen scale. The velocity $\boldsymbol{v}_{S3}$ of point S_3 is determined by the vectorial equations given by Eq.1 and Eq. 2:

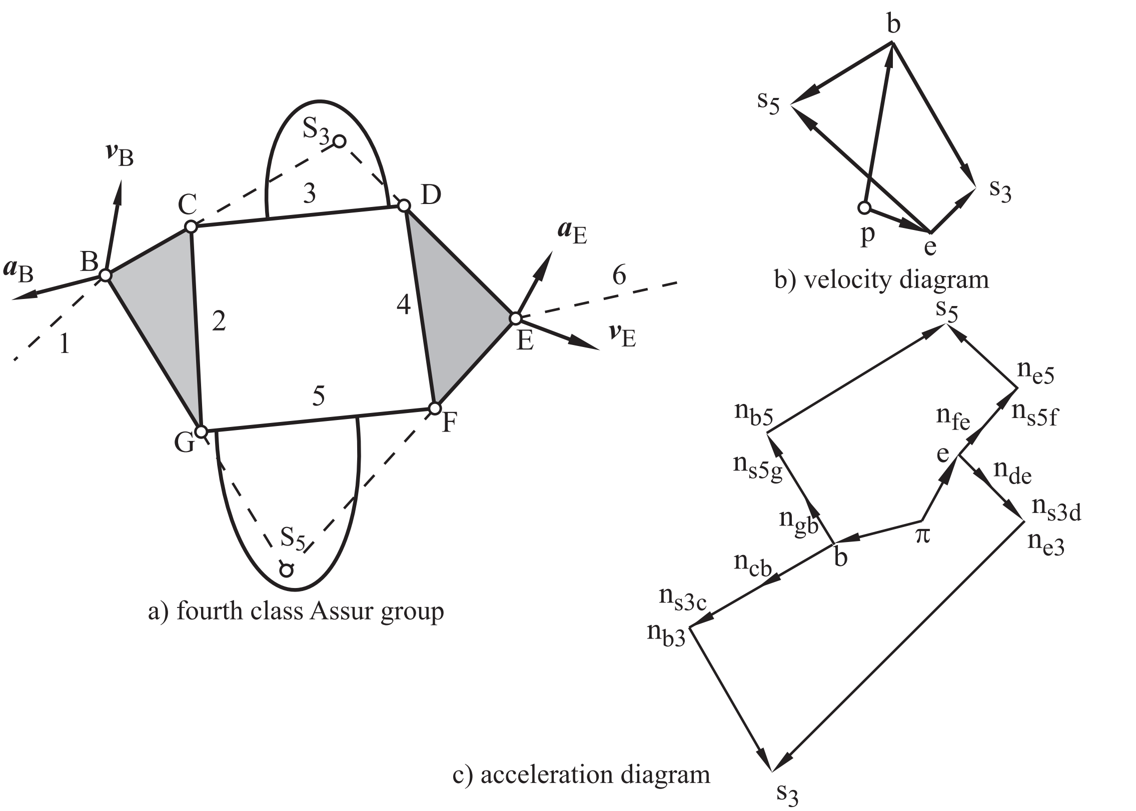

Figure 1 Velocity and acceleration diagram of a fourth class Assur group

$$\boldsymbol{v}_{S3} = \boldsymbol{v}_C + \boldsymbol{v}_{S3C} = \boldsymbol{v}_B + \overline{\boldsymbol{v}_{CB} + \boldsymbol{v}_{S3C}} \quad (1)$$

$$\boldsymbol{v}_{S3} = \boldsymbol{v}_D + \boldsymbol{v}_{S3D} = \boldsymbol{v}_E + \overline{\boldsymbol{v}_{DE} + \boldsymbol{v}_{S3D}} \quad (2)$$

The last two vectors of each equation can be represented by the same line because both vectors are perpendicular to S_3B and S_3E, respectively; therefore, vector $\boldsymbol{v}_{S3B} = \overline{\boldsymbol{v}_{CB} + \boldsymbol{v}_{S3C}}$ is perpendicular to line S_3B, and vector $\boldsymbol{v}_{S3E} = \overline{\boldsymbol{v}_{DE} + \boldsymbol{v}_{S3D}}$ is perpendicular to line S_3E.

Similarly, the vectorial equation for S_3 can be formulated using Eq.3 and Eq. 4:

$$\boldsymbol{v}_{S3} = \boldsymbol{v}_B + \boldsymbol{v}_{S3B} \quad (3)$$

$$\boldsymbol{v}_{S3} = \boldsymbol{v}_E + \boldsymbol{v}_{S3E} \quad (4)$$

For representing graphically Eq. 3 and Eq. 4, a straight line can be drawn from point *b* on the velocity diagram, with the direction of the velocity $\boldsymbol{v}_{S3B}$ (perpendicular to S_3B). Similarly, from point *e,* a straight line can be drawn with the direction of the velocity $\boldsymbol{v}_{S3E}$ (vector that is perpendicular to S_3E). The intersection point of these two straight lines on the velocity diagram (point s_3 from figure 1.b) represents the end of the vector of the velocity $\boldsymbol{v}_{S3}$ of point S_3. The magnitude of the velocity of this point can be obtained by multiplying the distance between points *p* and s_3 by the velocity scale factor chosen, μ_v:

$$v_{S3} = \mu_v (ps_3) \quad (5)$$

From the velocity diagram, it is possible to determine the relative velocities $\boldsymbol{v}_{S3B}$ and $\boldsymbol{v}_{S3E}$:

$$v_{S3B} = \mu_v (bs_3); \quad v_{S3E} = \mu_v (es_3); \quad (6)$$

These relative velocities depend on the angular velocities of the links that constitute the fourth class Assur group. To establish the corresponding equations, it is necessary to assume the directions for the angular velocities. The following relations can be obtained from the velocity diagram, shown in figure 2.b, and by assuming that the angular velocities of links 2 to 5 are clockwise.

$$\omega_2 L_{CB} + \omega_3 L_{S3C} = \mu_v (bs_3) \quad (7)$$

$$\omega_4 L_{DE} + \omega_3 L_{S3D} = \mu_v (es_3) \quad (8)$$

In Eq. 7, the direction of velocity $\boldsymbol{v}_{S3B}$ is the same as the direction obtained if the angular velocities of links 2 and 3 were clockwise; therefore, the resultant velocity is positive. In the same way, the relative velocity $\boldsymbol{v}_{S3E}$ of Eq. 8 has the same direction to the one obtained if the angular velocities for links 3 and 4 were clockwise.

The velocity $\boldsymbol{v}_{S5}$ of point S_5, related to link 5, is determined by the formulation of the relative velocity equation with respect to points G and F, Eq. 9 and Eq. 10:

$$\boldsymbol{v}_{S5} = \boldsymbol{v}_G + \boldsymbol{v}_{S5G} = \boldsymbol{v}_B + \overline{\boldsymbol{v}_{GB} + \boldsymbol{v}_{S5G}} \quad (9)$$

$$\boldsymbol{v}_{S5} = \boldsymbol{v}_F + \boldsymbol{v}_{S5F} = \boldsymbol{v}_E + \overline{\boldsymbol{v}_{FE} + \boldsymbol{v}_{S5F}} \quad (10)$$

The last two vectors of each equation are located on the same line, because both vectors are perpendicular to S_5B or to S_5E; this means that vector $\boldsymbol{v}_{S5B} = \overline{\boldsymbol{v}_{GB} + \boldsymbol{v}_{S5G}}$ is perpendicular to S_5B and vector $\boldsymbol{v}_{S5E} = \overline{\boldsymbol{v}_{FE} + \boldsymbol{v}_{S5F}}$ is perpendicular to S_5E.

Similarly to the velocity vectorial equations for point S_3, the vectorial equation system for S_5 can be described through Eqs.11 and 12:

$$\boldsymbol{v}_{S5} = \boldsymbol{v}_B + \boldsymbol{v}_{S5B} \quad (11)$$

$$\boldsymbol{v}_{S5} = \boldsymbol{v}_E + \boldsymbol{v}_{S5E} \quad (12)$$

For the graphical representation of these equations, figure 1.b, a straight line can be drawn from point *b* on the velocity diagram with the direction of velocity $\boldsymbol{v}_{S5B}$ (perpendicular to S_5B).

Similarly, a straight line is drawn from point *e* with the direction of the vector for the velocity $\boldsymbol{v}_{S5E}$ (perpendicular to S_5E). The intersection point of these two straight lines on the velocity diagram (point s_5 in figure 1.b) represents the end of the vector for the velocity $\boldsymbol{v}_{S5}$ of point S_5. The magnitude of the velocity of this point can be obtained by multiplying the length of line ps_5 by the velocity scale factor:

$$v_{S5} = \mu_v (ps_5) \tag{13}$$

Then, from the velocity diagram, it is possible to determine the relative velocities $\boldsymbol{v}_{S5B}$ and $\boldsymbol{v}_{S5E}$

$$v_{S5B} = \mu_v (bs_5); \quad v_{S5E} = \mu_v (es_5); \tag{14}$$

Similar to the analysis of the velocity diagram of point S_3, the relative velocities $\boldsymbol{v}_{S5B}$ and $\boldsymbol{v}_{S5E}$ depend also on the direction of the angular velocities for links 2, 3, 4, and 5. Considering all the angular velocities clockwise, the following equations are given:

$$\omega_2 L_{GB} + \omega_5 L_{S5G} = \mu_v (bs_5) \tag{15}$$

$$\omega_4 L_{FE} + \omega_5 L_{S5F} = \mu_v (es_5) \tag{16}$$

In both equations, the relative velocities are positive because the directions of velocities $\boldsymbol{v}_{S3B}$ and $\boldsymbol{v}_{S3E}$ are the same as the direction obtained if the angular velocities for links 2, 3, 4, and 5 were clockwise. Solving the system of linear equation given by Eqs. 7, 8, 15, and 16 simultaneously, the angular velocities for the links 2, 3, 4, and 5 are determined.

Analysis of accelerations

The solution to the analysis of accelerations for a fourth class Assur group with two drag members can be obtained in a similar manner to the velocities solution. As in the analysis for velocities, it is necessary to make use of the special points S_3, related to link 3, and S_5, related to link 5.

Taking an arbitrary point π (figure 1.c) as a pole and as a starting point for segments *πb* and *πe* for the accelerations diagram, the segments *πb* and *πe* can be drawn; these segments represent the accelerations $\boldsymbol{a}_B$ and $\boldsymbol{a}_E$ at the chosen scale μ_a for points B and E. The acceleration $\boldsymbol{a}_{S3}$ of the special point S_3 can be determined by using Eqs. 17 and 18:

$$\begin{aligned} \boldsymbol{a}_{S_3} &= \boldsymbol{a}_C + \left(\boldsymbol{a}^n_{S_3C} + \boldsymbol{a}^t_{S_3C}\right) = \boldsymbol{a}_B + \left(\boldsymbol{a}^n_{CB} + \boldsymbol{a}^t_{CB}\right) + \left(\boldsymbol{a}^n_{S_3C} + \boldsymbol{a}^t_{S_3C}\right) \\ \boldsymbol{a}_{S_3} &= \boldsymbol{a}_B + \overline{\boldsymbol{a}^n_{CB} + \boldsymbol{a}^n_{S_3C}} + \overline{\boldsymbol{a}^t_{CB} + \boldsymbol{a}^t_{S_3C}} = \boldsymbol{a}_B + \boldsymbol{a}^n_{S_3B} + \boldsymbol{a}^t_{S_3B} \end{aligned} \tag{17}$$

$$\begin{aligned} \boldsymbol{a}_{S_3} &= \boldsymbol{a}_D + \left(\boldsymbol{a}^n_{S_3D} + \boldsymbol{a}^t_{S_3D}\right) = \boldsymbol{a}_E + \left(\boldsymbol{a}^n_{DE} + \boldsymbol{a}^t_{DE}\right) + \left(\boldsymbol{a}^n_{S_3D} + \boldsymbol{a}^t_{S_3D}\right) \\ \boldsymbol{a}_{S_3} &= \boldsymbol{a}_E + \overline{\boldsymbol{a}^n_{DE} + \boldsymbol{a}^n_{S_3D}} + \overline{\boldsymbol{a}^t_{DE} + \boldsymbol{a}^t_{S_3D}} = \boldsymbol{a}_E + \boldsymbol{a}^n_{S_3E} + \boldsymbol{a}^t_{S_3E} \end{aligned} \tag{18}$$

In Eqs. 17 and 18, the sums of the normal and tangential accelerations are shown as vectors $\boldsymbol{a}^n_{S_3B}$, $\boldsymbol{a}^n_{S_3E}$, $\boldsymbol{a}^t_{S_3B}$ and $\boldsymbol{a}^t_{S_3E}$, because the directions of their components are the same. The magnitudes for the relative normal accelerations are determined by:

$$\begin{aligned} a^n_{CB} &= v^2_{CB} / L_{CB} \qquad & a^n_{S_3C} &= v^2_{S_3C} / L_{CS_3} \\ a^n_{DE} &= v^2_{DE} / L_{DE} \qquad & a^n_{S_3D} &= v^2_{S_3D} / L_{DS_3} \end{aligned} \tag{19}$$

The directions for these vectors can also be determined by the same methods already mentioned. As the direction of both tangential components, for each vectorial equation, are the same, it is not necessary to determine the magnitude of each components. To calculate the resultant of the sum of the tangential components, drawing vector action line for the tangential accelerations from the ends of the normal accelerations $\boldsymbol{a}^n_{S_3B}$, $\boldsymbol{a}^n_{S_3E}$ is sufficcient. For this purpose, segments bn_{b3} and en_{e3} can be drawn from points *b* and *e,* obtained from the acceleration diagram; these are the representations for the accelerations $\boldsymbol{a}^n_{S_3B}$ and $\boldsymbol{a}^n_{S_3E}$ (at scale μ_a).

Then, straight lines are drawn in the direction of the tangential accelerations $\boldsymbol{a}^t_{S_3B}$ and $\boldsymbol{a}^t_{S_3E}$ from points n_{b3} and n_{e3}, which are perpendicular to lines S_3B and S_3E, respectively. Point s_3, where these two lines intersect, is the end of vector $\boldsymbol{a}_{S3}$; the magnitude of the absolute acceleration of point S_3 is determined by:

$$a_{S3} = \mu_a(\pi\ s_3) \tag{20}$$

From the acceleration diagram, it is now possible to determine the magnitude of the relative accelerations $\boldsymbol{a}^{t}_{S_3B}$ and $\boldsymbol{a}^{t}_{S_3E}$:

$$a^{t}_{S_3B} = \mu_a(n_{b3}s_3); \quad a^{t}_{S_3E} = \mu_a(n_{e3}s_3); \tag{21}$$

The senses for the relative tangential accelerations and the angular accelerations follow the same procedure described for the velocity case. These relative tangential accelerations depend on the angular accelerations of the links that constitute the fourth class group. For this case, all the angular accelerations are assumed clockwise. The following relations are obtained from the accelerations diagram:

$$\alpha_2 L_{CB} + \alpha_3 L_{S3C} = \mu_a(n_{b3}s_3) \tag{22}$$

$$\alpha_4 L_{DE} + \alpha_3 L_{S3D} = -\mu_a(n_{e3}s_3) \tag{23}$$

The tangential acceleration is positive in the former equation, because the direction of the acceleration $\boldsymbol{a}^{t}_{S_3B}$ is the same as the one that the angular accelerations for links 2 and 3 would have if they were clockwise. In the second case, the relative tangential acceleration $\boldsymbol{a}^{t}_{S_3E}$ has opposite direction to the one that would be obtained if the angular accelerations for links 3 and 4 were clockwise; this is the reason for the negative sign.

The acceleration $\boldsymbol{a}_{S5}$ from the special point S_5 is determined by using Eqs. 24 and 25:

$$\begin{aligned} \boldsymbol{a}_{S_5} &= \boldsymbol{a}_G + \left(\boldsymbol{a}^{n}_{S_5G} + \boldsymbol{a}^{t}_{S_5G}\right) = \boldsymbol{a}_B + \left(\boldsymbol{a}^{n}_{GB} + \boldsymbol{a}^{t}_{GB}\right) + \left(\boldsymbol{a}^{n}_{S_5G} + \boldsymbol{a}^{t}_{S_5G}\right) \\ \boldsymbol{a}_{S_5} &= \boldsymbol{a}_B + \overline{\boldsymbol{a}^{n}_{GB} + \boldsymbol{a}^{n}_{S_5G}} + \overline{\boldsymbol{a}^{t}_{GB} + \boldsymbol{a}^{t}_{S_5G}} = \boldsymbol{a}_B + \boldsymbol{a}^{n}_{S_5B} + \boldsymbol{a}^{t}_{S_5B} \end{aligned} \tag{24}$$

$$\begin{aligned} \boldsymbol{a}_{S_5} &= \boldsymbol{a}_F + \left(\boldsymbol{a}^{n}_{S_5F} + \boldsymbol{a}^{t}_{S_5F}\right) = \boldsymbol{a}_E + \left(\boldsymbol{a}^{n}_{FE} + \boldsymbol{a}^{t}_{FE}\right) + \left(\boldsymbol{a}^{n}_{S_5F} + \boldsymbol{a}^{t}_{S_5F}\right) \\ \boldsymbol{a}_{S_5} &= \boldsymbol{a}_E + \overline{\boldsymbol{a}^{n}_{FE} + \boldsymbol{a}^{n}_{S_5F}} + \overline{\boldsymbol{a}^{t}_{FE} + \boldsymbol{a}^{t}_{S_5F}} = \boldsymbol{a}_E + \boldsymbol{a}^{n}_{S_5E} + \boldsymbol{a}^{t}_{S_5E} \end{aligned} \tag{25}$$

In Eqs. 24 and 25 the sums of the normal and tangential accelerations are shown as resultant vectors $\boldsymbol{a}^{n}_{S_5B}$, $\boldsymbol{a}^{n}_{S_5E}$, $\boldsymbol{a}^{t}_{S_5B}$, and $\boldsymbol{a}^{t}_{S_5E}$, as the directions of their components are equal. The magnitudes of the normal accelerations are determined as usual, making use of Eq. 26:

$$\begin{aligned} a^{n}_{GB} &= v^2_{GB}/L_{GB} \qquad & a^{n}_{S_5G} &= v^2_{S_5G}/L_{GS_5} \\ a^{n}_{FE} &= v^2_{FE}/L_{FE} \qquad & a^{n}_{S_5F} &= v^2_{S_5F}/L_{FS_5} \end{aligned} \tag{26}$$

The direction of the normal acceleration vectors is also determined by the same methods already mentioned. In the acceleration diagram, tangential accelerations are drawn from the ends of the normal accelerations $\boldsymbol{a}^{n}_{S_5B}$ and $\boldsymbol{a}^{n}_{S_5E}$. The directions of these vectors are perpendicular to the vectors of the normal accelerations. For this purpose, segments bn_{b5} and en_{e5} are drawn starting on points b and e. These segments are obtained from the acceleration diagram and are the representations for the accelerations $\boldsymbol{a}^{n}_{S_5B}$ and $\boldsymbol{a}^{n}_{S_5E}$.

Then, straight lines are drawn from points n_{b3} and n_{e3} in the direction of the accelerations $\boldsymbol{a}^{t}_{S_5B}$ and $\boldsymbol{a}^{t}_{S_5E}$, which are perpendicular to S_5B and S_5E respectively. Point s_5, which is the intersection point of these two lines, is the end of vector $\boldsymbol{a}_{S5}$ from the resultant acceleration of point S_5, whose magnitude is determined by:

$$a_{S_5} = \mu_a(\pi\ s_5) \tag{27}$$

Repeating the procedure for the relative accelerations of point S_3, the magnitude of the relative accelerations $\boldsymbol{a}^{t}_{S_5B}$ and $\boldsymbol{a}^{t}_{S_5E}$ are determined from the acceleration diagram, multiplying by the acceleration scale factor

$$a^{t}_{S_5B} = \mu_a(n_{b5}s_5); \quad a^{t}_{S_5E} = \mu_a(n_{e5}s_5); \tag{28}$$

Now, these relative tangential accelerations depend on the angular accelerations of the links that constitute the fourth class group. Taking into account that all the angular accelerations are assumed clockwise, the following relations are obtained from the accelerations diagram:

$$\alpha_2 L_{GB} + \alpha_5 L_{S5G} = -\mu_a(n_{b5}s_5) \tag{29}$$

$$\alpha_4 L_{FE} + \alpha_5 L_{S5F} = \mu_a (n_{e5} s_5) \quad (30)$$

In Eqs. 29 the relative tangential accelerations $\boldsymbol{a}^{t}_{S_5B}$ are opposite to one that angular accelerations of links 2 and 5 were clockwise. For that reason, it is necessary to write the negative sign in Eqs. 29.

From the simultaneous solution of the systems of linear equation given by Eqs. 22, 23, 29, and 30, the angular accelerations for the links that constitute the fourth class group can be determined.

Results

In the present section, a mechanism R – (RRP – RRR) of 1 DOF is considered as example. Figure 2.a shows the representation of a mechanism comprised by driving link 1, fixed link 6, and a fourth class group (links 2, 3, 4, and 5). Links 2 and 4 are the ternary links of the group, and joints B and G are the joints that connect the group to the base mechanism. Links 3 and 5 join to the two drag links.

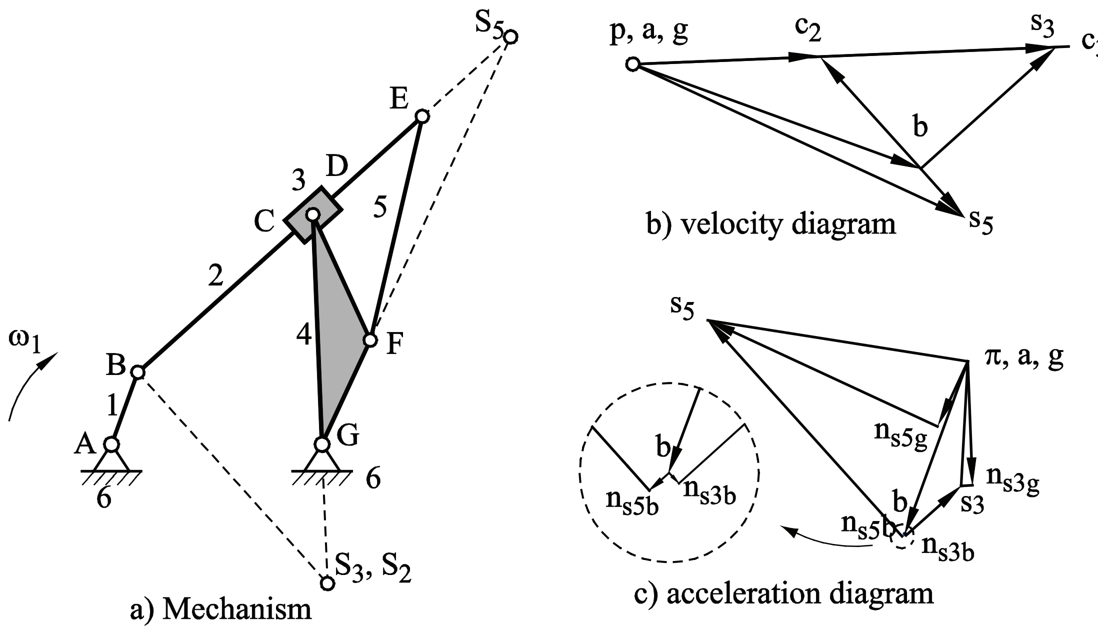

Figure 2 Example of the mechanism of study

The lengths of the links are: L_{AG} = 275 mm, L_{AB} = 100 mm, L_{BE} = 500 mm, L_{EF} = 300 mm, L_{GD} = 300 mm, L_{FG} = 150 mm, and L_{FD} = 180 mm; the angular velocity of link 1 is ω_1 = 1 rad/s. The additional lengths required for the analysis of velocities and accelerations are obtained from the positions diagram. The additional lengths are the distances measured from the joints to the special points S_3 and S_5: L_{BS3} = 248.0601 mm, L_{DS3} = 481.9848 mm, L_{FS5} = 436.2923 mm, and L_{ES5} = 155.1123 mm.

For the kinematic analysis of the fourth class Assur group, it is necessary to determine the special point S_3, which belongs to link 3, and point S_5, which belongs to link 5. Link 3 has a prismatic joint with link 2 and one rotation joint with link 4. The procedure to locate the special point requires a modification with respect to the case in which the link with internal joints has only rotation joints. The special point of link 3 must be such that, when formulating the relative velocities equation, the unknown velocities are parallel. Observing that the relative movement between links 2 and 3 is parallel to link 3, the special point must be located on a line that is perpendicular to link 2 and that intercepts point B. Point S_3 is located on the intersection between a line that is perpendicular to the relative movement between links 2 and 3 and the projection of line GD of link 4; point D represents the rotation joint between links 4 and 3. Point S_2 is the point of link 2 concurrent to point S_3.

The binary link 5 joins with the ternary links (2 and 4) with rotation joints. Point S_5 is the intersection point when line BE related to link 2 and line GF related to link 4 are projected.

***Analysis of velocities**:* Figure 2.b shows the velocities diagram of the mechanism.

The solution for the driving link is:

$$\boldsymbol{v}_{\mathrm{B}} = \boldsymbol{v}_{\mathrm{A}} + \boldsymbol{v}_{\mathrm{BA}} \quad (31)$$

$$\boldsymbol{v}_{\mathrm{BA}} = \omega_1 \cdot L_{\mathrm{BA}} = 1\,\mathrm{rad/s} \cdot 100\,\mathrm{mm} = 10\,\mathrm{mm/s} \quad (32)$$

The equations for the velocity of point S_3 related to link 3 are obtained using the equation of relative velocities from joints S_2 and D:

$$\begin{aligned} \boldsymbol{v}_{\mathrm{S3}} &= \boldsymbol{v}_{\mathrm{S2}} + \boldsymbol{v}_{\mathrm{S3S2}} = (\boldsymbol{v}_{\mathrm{B}} + \boldsymbol{v}_{\mathrm{S2B}}) + \boldsymbol{v}_{\mathrm{S3S2}} = \boldsymbol{v}_{\mathrm{B}} + \overline{v_{\mathrm{S2B}} + v_{\mathrm{S3S2}}} \\ \boldsymbol{v}_{\mathrm{S3}} &= \boldsymbol{v}_{\mathrm{D}} + \boldsymbol{v}_{\mathrm{S3D}} = (\boldsymbol{v}_{\mathrm{G}} + \boldsymbol{v}_{\mathrm{DG}}) + \boldsymbol{v}_{\mathrm{S3D}} = \boldsymbol{v}_{\mathrm{G}} + \overline{v_{\mathrm{DG}} + v_{\mathrm{S3D}}} \end{aligned} \quad (33)$$

The velocities v_{S2B} and v_{S3S2} are parallel, because v_{S2B} is perpendicular to line BS_2, and v_{S3S2} is parallel to link 2. As BS_2 is perpendicular to link 2, they can be drawn together as is presented in Eq. 33. A similar procedure is given for velocities v_{DG} and v_{S3G}, which are perpendicular to line DG.

For link 5, special point S_5 has to be determined, by projecting lines BE and GF. The procedure to determine the velocity of point S_5 is the same as the one followed for point S_3; that is, using relative velocities from points E and F:

$$\begin{aligned} \boldsymbol{v}_{\mathrm{S5}} &= \boldsymbol{v}_{\mathrm{E}} + \boldsymbol{v}_{\mathrm{S5E}} = (\boldsymbol{v}_{\mathrm{B}} + \boldsymbol{v}_{\mathrm{EB}}) + \boldsymbol{v}_{\mathrm{S5E}} = \boldsymbol{v}_{\mathrm{B}} + \overline{v_{\mathrm{EB}} + v_{\mathrm{S5E}}} \\ \boldsymbol{v}_{\mathrm{S5}} &= \boldsymbol{v}_{\mathrm{F}} + \boldsymbol{v}_{\mathrm{S5F}} = (\boldsymbol{v}_{\mathrm{G}} + \boldsymbol{v}_{\mathrm{FG}}) + \boldsymbol{v}_{\mathrm{S5F}} = \boldsymbol{v}_{\mathrm{G}} + \overline{v_{\mathrm{FG}} + v_{\mathrm{S5F}}} \end{aligned} \quad (34)$$

Point S_5 is chosen so that the velocities v_{EB} and v_{S5E} become parallel, as v_{FG} and $v_{S5F.}$

Figure 2.b shows the velocities diagram, which is useful to solve the velocities at points S_3 and S_5. The distance between points s_3 and b (from the velocities diagram) gives the sum of the relative velocities $\boldsymbol{v}_{\mathrm{S2B}}$ and $\boldsymbol{v}_{\mathrm{S3S2}}$. Assuming clockwise angular velocities for links 2 and 3 the relative movement between links 2 and 3 from left to right (observed from link 2), the following equation results:

$$v_{\mathrm{S2B}} + v_{\mathrm{S3S2}} = -\omega_2 L_{\mathrm{BS2}} + v_{\mathrm{S3S2}} = \mu_v \cdot \overline{\mathrm{bs}_3} = 59.3891\,\mathrm{mm/s} \quad (35)$$

where the distance between points b and s_3 is represented by line $\overline{\mathrm{bs}_3}$; its magnitude is obtained by multiplying it by the scale actor μ_v. If the direction of the angular velocity of link 2 is assumed clockwise, then the direction of the relative velocity $\boldsymbol{v}_{\mathrm{S2B}}$ is opposite to the resultant velocity $\overline{\mathrm{bs}_3}$. A Similar procedure is followed for the other velocities:

$$v_{\mathrm{DG}} + v_{\mathrm{S3D}} = \omega_4 L_{\mathrm{DG}} - \omega_3 L_{\mathrm{DS3}} = \mu_v \cdot \overline{\mathrm{gs}_3} = 138.2264\,\mathrm{mm/s} \quad (36)$$

If the angular velocities of links 3 and 4 are assumed clockwise, the relative velocities v_{DG} and v_{S3D} have opposite directions. Then, in Eq. 36, the latter relative velocity v_{S3D} is negative. Next, the equations for the relative velocities related to point S_5 can be formulated as:

$$v_{\mathrm{EB}} + v_{\mathrm{S5E}} = \omega_2 L_{\mathrm{EB}} + \omega_5 L_{\mathrm{ES5}} = \mu_v \cdot \overline{\mathrm{bs}_5} = 21.4233\,\mathrm{mm/s} \quad (37)$$

$$v_{\mathrm{FG}} + v_{\mathrm{S5F}} = \omega_4 L_{\mathrm{FG}} + \omega_5 L_{\mathrm{FS5}} = \mu_v \cdot \overline{\mathrm{gs}_5} = 119{,}3355\,\mathrm{mm/s} \quad (38)$$

In the linear equation systems given by Eqs. 35, 36, 37 and 38, the unknown variables are the angular velocities ω_2, ω_4, and ω_5 and the relative velocity between links 2 and 3, v_{S3S2}. This system of equations can now be solved. The distances L_{BS3}, L_{ES5}, and L_{FS5} can be taken from the positions diagram. The solution for the system of equations gives the following results:

$\omega_2 = 0.0086$ rad/s, $\omega_4 = 0.4746$ rad/s, $\omega_5 = 0.1104$ rad/s, $v_{S3S2} = 61.5254$ mm/s.

Analysis for accelerations**:** Figure 2.c represents the acceleration diagram for the mechanism.

The first step consists of solving the driving link.

$$\boldsymbol{a}_{B} = \boldsymbol{a}_{A} + \boldsymbol{a}_{BA}^{n} + \boldsymbol{a}_{BA}^{t} \quad (39)$$

$$\boldsymbol{a}_{A} = \boldsymbol{a}_{BA}^{t} = 0 \quad (40)$$

$$a_{BA}^{n} = \omega_1^2 \cdot L_{AB} = (1\ \text{rad/s})^2 \cdot 100\ \text{mm} = 100\ \text{mm/s}^2 \quad (41)$$

The direction of the acceleration a_{BA}^{n} is from point B to point A. Next, the accelerations of point S_3, related to link 3, and S_5, related to link 5, are determined.

$$\boldsymbol{a}_{S3} = \boldsymbol{a}_{S2} + \boldsymbol{a}_{S3S2}^{c} + \boldsymbol{a}_{S3S2}^{\rho} = \left(\boldsymbol{a}_{B} + \boldsymbol{a}_{S2B}^{n} + \boldsymbol{a}_{S2B}^{t}\right) + \boldsymbol{a}_{S3S2}^{c} + \boldsymbol{a}_{S3S2}^{\rho} \quad (42)$$

where $\boldsymbol{a}_{S2B}^{n} = \omega_2^2 \cdot L_{BS2} = (0.0086\,\text{rad/s})^2 \cdot 248.0601\,\text{mm} = 0.0183\,\text{mm/s}^2$, the direction is from point D to E

$\boldsymbol{a}_{S2B}^{t} = \alpha_2 \cdot L_{S2B}$, that is perpendicular to line BS_2

$\boldsymbol{a}_{S3S2}^{c} = 2\omega_2 \cdot v_{S3S2} = 2 \cdot 0.0086\,\text{rad/s} \cdot 61.5254\,\text{mm/s} = 1.0582\,\text{mm/s}^2$ the direction of this component is obtained by rotating the vector v_{S3S2} through of angle of 90° in the direction of the angular velocity ω_2 (the direction is opposite to that of the component $\boldsymbol{a}_{S2B}^{n}$).

$\boldsymbol{a}_{S3D}^{\rho}$ is parallel to link 2.

In Eq. 42, the accelerations $\boldsymbol{a}_{DE}^{t}$ and $\boldsymbol{a}_{S3S2}^{\rho}$ are unknown. The components $\boldsymbol{a}_{S2B}^{n}$ and $\boldsymbol{a}_{S3S2}^{c}$ are parallel due to the method to find point S_3 (this point is located on line DE). A similar case occurs with the components $\boldsymbol{a}_{DE}^{t}$ and $\boldsymbol{a}_{S3S2}^{\rho}$. Making use of this condition, Eq.42 can be formulated as:

$$\boldsymbol{a}_{S3} = \boldsymbol{a}_{B} + \overline{a_{S3S2}^{c} - a_{S2B}^{n}} + \overline{a_{S2B}^{t} + a_{S3S2}^{\rho}} = \boldsymbol{a}_{B} + \boldsymbol{a}_{S3B}^{n} + \boldsymbol{a}_{S3B}^{t} \quad (43)$$

Figure 2.c shows the representation for $\boldsymbol{a}_{S2B}^{n} + \boldsymbol{a}_{S3S2}^{c}$ through line bn_{s3b}. As this component has opposite direction, its total magnitude can be obtained by subtracting both components in the direction of line bs_3. The resultant tangential acceleration $\boldsymbol{a}_{S3B}^{t}$ (perpendicular to the normal acceleration) is represented through point n_{s3b}. A second equation is determined for point S_3:

$$\boldsymbol{a}_{S3} = \boldsymbol{a}_{D} + \boldsymbol{a}_{S3D}^{n} + \boldsymbol{a}_{S3D}^{t} = \left(\boldsymbol{a}_{G} + \boldsymbol{a}_{DG}^{n} + \boldsymbol{a}_{DG}^{t}\right) + \boldsymbol{a}_{S3D}^{n} + \boldsymbol{a}_{S3D}^{t} \quad (44)$$

where

$\boldsymbol{a}_{DG}^{n} = \omega_4^2 \cdot L_{DG} = (0.4746\ \text{rad/s})^2 \cdot 300\,\text{mm} = 67.5735\,\text{mm/s}^2$, from D to G

$\boldsymbol{a}_{DG}^{t} = \alpha_4 \cdot L_{DG}$, that is perpendicular to DG

$\boldsymbol{a}_{S3D}^{n} = \omega_3^2 \cdot L_{S3D} = (0{,}0086\ \text{rad/s})^2 \cdot 481{,}9848\,\text{mm} = 0{,}0356\,\text{mm/s}^2$, from S3 to D

$\boldsymbol{a}_{S3D}^{t} = \alpha_3 \cdot L_{S3D}$, perpendicular to line S_3D

In this equation, the tangential accelerations are unknown. Due to the method used to find point S_3 (this point is located on line DG), the normal components $\boldsymbol{a}_{DG}^{n}$ and $\boldsymbol{a}_{S3D}^{n}$ and the tangential components $\boldsymbol{a}_{DG}^{t}$ and $\boldsymbol{a}_{S3D}^{t}$ are parallel. Making use of this condition, Eq. 44 can be expressed as:

$$\boldsymbol{a}_{S3} = \boldsymbol{a}_{G} + \boldsymbol{a}_{DG}^{n} + \boldsymbol{a}_{S3D}^{n} + \overline{a_{DG}^{t} + a_{S3D}^{t}} = \boldsymbol{a}_{G} + \boldsymbol{a}_{S_3G}^{n} + \boldsymbol{a}_{S_3G}^{t} \quad (45)$$

The normal acceleration $\boldsymbol{a}_{S_3G}^{n}$ is represented through the segment gn_{s3g} on the accelerations diagram. Here, the normal accelerations are subtracted by each other and the resultant acceleration can be found. The component $\boldsymbol{a}_{S_3G}^{t}$ is represented through a line drawn from point n_{s3e}, perpendicular to the normal component $\boldsymbol{a}_{S_3G}^{n}$. The point of intersection of the lines that represent the relative tangential accelerations is the point of the acceleration of point S_3, $\boldsymbol{a}_{S3}$.

Two equations are needed to find the acceleration S_5 (S_5 belongs to link 5). These equations are taken from the relative accelerations between points S_5 and E, and between points S_5 and F:

$$\boldsymbol{a}_{S5} = \boldsymbol{a}_{F} + \boldsymbol{a}_{S5F}^{n} + \boldsymbol{a}_{S5F}^{t} = \left(\boldsymbol{a}_{G} + \boldsymbol{a}_{FG}^{n} + \boldsymbol{a}_{FG}^{t}\right) + \boldsymbol{a}_{S5F}^{n} + \boldsymbol{a}_{S5F}^{t} \quad (46)$$

where

$\boldsymbol{a}_{FG}^{n} = \omega_4^2 \cdot L_{FG} = (0.4746\,\text{rad/s})^2 \cdot 150\,\text{mm} = 33.7869\,\text{mm/s}^2$, the direction of this acceleration is given by line FG

$\boldsymbol{a}_{FG}^{t} = \alpha_4 \cdot L_{FG}$, that is perpendicular to line FG

$\boldsymbol{a}^{n}_{S5F} = \omega_5^2 \cdot L_{S5G} = (0.1104\ \text{rad/s})^2 \cdot 436.2923\ \text{mm} = 5.3176\ \text{mm/s}^2$, with direction from S_5 to F

$\boldsymbol{a}^{t}_{S5F} = \alpha_5 \cdot L_{S5F}$, that is perpendicular to line S_5F

Due to the parallelism condition, Eq. 46 can be reduced to:

$$\boldsymbol{a}_{S5} = \boldsymbol{a}_{G} + \boldsymbol{a}^{n}_{FG} + \boldsymbol{a}^{n}_{S5F} + \overline{a^{t}_{FG} + a^{t}_{S5F}} = \boldsymbol{a}_{G} + \boldsymbol{a}^{n}_{S5G} + \boldsymbol{a}^{t}_{S5G} \quad (47)$$

where $\boldsymbol{a}^{n}_{S5G} = \boldsymbol{a}^{n}_{FG} + \boldsymbol{a}^{n}_{S5F}$ is represented through line gn_{s5g}; the magnitude is obtained by adding the normal components from F to G, and the tangential acceleration $\boldsymbol{a}^{t}_{S5G} = \overline{a^{t}_{FG} + a^{t}_{S5F}}$ is represented from point n_{s5g}, and it is perpendicular to the direction of the normal acceleration.

Taking relative accelerations between points E and S_5:

$$\boldsymbol{a}_{S5} = \boldsymbol{a}_{E} + \boldsymbol{a}^{n}_{S5E} + \boldsymbol{a}^{t}_{S5EF} = (\boldsymbol{a}_{B} + \boldsymbol{a}^{n}_{EB} + \boldsymbol{a}^{t}_{EB}) + \boldsymbol{a}^{n}_{S5E} + \boldsymbol{a}^{t}_{S5E} \quad (48)$$

where $\boldsymbol{a}^{n}_{EB} = \omega_2^2 \cdot L_{EB} = (0.0086\ \text{rad/s})^2 \cdot 500\ \text{mm} = 0.037\ \text{mm/s}^2$, from E to B

$\boldsymbol{a}^{t}_{EB} = \alpha_2 \cdot L_{EB}$, perpendicular to line EB

$\boldsymbol{a}^{n}_{S5E} = \omega_5^2 \cdot L_{S5E} = (0.1104\ \text{rad/s})^2 \cdot 155.1123\ \text{mm} = 1.8905\ \text{mm/s}^2$, from S_5 to E

$\boldsymbol{a}^{t}_{S5E} = \alpha_5 \cdot L_{S5E}$, perpendicular to line S_5E

Making use of the parallelism condition, Eq. 48 can be formulated as:

$$\boldsymbol{a}_{S5} = \boldsymbol{a}_{B} + \boldsymbol{a}^{n}_{EB} + \boldsymbol{a}^{n}_{S5E} + \overline{a^{t}_{EB} + a^{t}_{S5E}} = \boldsymbol{a}_{B} + \boldsymbol{a}^{n}_{S5B} + \boldsymbol{a}^{t}_{S5B} \quad (49)$$

where $\boldsymbol{a}^{n}_{S5B} = \boldsymbol{a}^{n}_{EB} + \boldsymbol{a}^{n}_{S5E}$ is represented through line bn_{s5b}; the magnitude is obtained by adding the normal components from E to B, and the tangential acceleration $\boldsymbol{a}^{t}_{S5B} = \overline{a^{t}_{EB} + a^{t}_{S5B}}$ is represented from point n_{s5b}, and it is perpendicular to the direction of the normal acceleration.

Figure 2.c represents the graphical procedure for the accelerations of points S_3 and S_5. The expressions for the relative tangential accelerations are obtained from the accelerations diagram. To determine the equations, the angular accelerations for links 2, 3, 4, and 5 are assumed to be counterclockwise, and the relative acceleration $\boldsymbol{a}^{\rho}_{S3S2}$ from left to right.

$$a^{t}_{EB} + a^{t}_{S5E} = \alpha_2 L_{EB} + \alpha_5 L_{S5E} = \mu_a \cdot \overline{n_{s5b}s_5} = 158.2219\ \text{mm/s}^2 \quad (50)$$

$$a^{t}_{FG} + a^{t}_{S5F} = \alpha_4 L_{FG} + \alpha_5 L_{S5F} = \mu_a \cdot \overline{n_{s5g}s_5} = 137.7597\ \text{mm/s}^2 \quad (51)$$

$$a^{t}_{S2B} + a^{\rho}_{S3S2} = \alpha_2 L_{S2B} + a^{\rho}_{S3S2} = \mu_a \cdot \overline{n_{s3b}s_3} = 40.3939\ \text{mm/s}^2 \quad (52)$$

$$a^{t}_{DG} + a^{t}_{S3D} = \alpha_4 L_{DG} - \alpha_3 L_{S3D} = \mu_a \cdot \overline{n_{s5g}s_g} = 6.173\ \text{mm/s}^2 \quad (53)$$

In the system of linear equations given by Eqs. 50 to 53, it is necessary to analyze the directions of the tangential accelerations in order to assign the matching sign. In these set of equations, the variables are: the angular accelerations α_2, $\alpha_3 = \alpha_2$, α_4, and α_5 and the relative acceleration a^{ρ}_{S3S2}. Solving the set of equations yields:

$\alpha_2 = \alpha_3 = 0.2663\ \text{rad/s}^2$, $\alpha_4 = 0.4485\ \text{rad/s}^2$, $\alpha_5 = 0.1616\ \text{rad/s}^2$, and $a^{\rho}_{S3S2} = -25.6695\ \text{m/s}^2$

Conclusions

A method that uses a combination of analytical and graphical methods to perform the kinematic analysis of a fourth class Assur group was presented in this paper. Special Assur points, which are points that belong to the links with internal joints, are used in the proposed solution.

An example that comprised a mechanisms R – (RRR – RRR) was presented to show the application of the proposed method; the solution for this example was verified using a commercial software and classical analytical methods. The results obtained demonstrate the reliability of the proposed method.

The developed method can be used for the kinematic analysis of planar mechanisms with one, two, or three degrees of freedom, including such groups. This method allows developing a modular method for the kinematic analysis of mechanisms. This is especially appropriate for pedagogical purposes.

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