Mathematical formulation of restrictions for the design of low voltage bandgap references

Formulación matemática de restricciones para el diseño de referencias de banda prohibida de bajo voltaje

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(Recibido el 10 de agosto de 2009. Aceptado el 14 de abril de 2011)

Abstract

Nowadays the qualities more looked for in the design of bandgap references are the low potential of the reference and the low value of its coefficient of temperature, being these parameters more favorable in curvature corrected bandgap references.

The design parameters of bandgap references depend on the value of the potential of reference $V_{REF\,;}$ nevertheless, in the reported designs there are not paid attention to conditions that limit the attainable value of V_{REF}.

In this work the mathematical formulation of the restrictions that really limit the attainable value of the reference potential is presented in order that the design of a curvature corrected bandgap reference can be realizable.

----- *Keywords:* curvature correction, low temperature coefficient, low voltage bandgap references

Resumen

Actualmente las cualidades más buscadas de las referencias bandgap son el bajo potencial de referencia y el bajo valor del coeficiente de temperatura, siendo estos parámetros mas favorables en las referencias de bandgap con corrección de curvatura. Por otra parte y aunque claramente los parámetros de diseño dependen del valor del potencial de referencia V_{REF}. Habitualmente en los diseños reportados no se analizan las condiciones que limitan el valor de V_{REF} alcanzable.

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En este trabajo se presenta la formulación matemática de las restricciones que realmente limitan el valor alcanzable del potencial de referencia para que el diseño de una fuente de referencia de bandgap con corrección de curvatura sea realizable.

----- *Palabras clave:* bajo coeficiente de temperatura, corrección de curvatura, referencia bandgap de bajo potencial

Introduction

Bandgap voltage references are frequently used in analog circuits as voltage regulators and as such in A/D and D/A converters [1]. At present the qualities more looked for in the design of bandgap references are the low values of voltage and temperature coefficient [2,3].

Although the dependencies of design parameters on the wished value of reference voltage V_{REF} are known, in the designs reported in the literature [3,4] the attainable value of V_{REF} is rarely analyzed or its importance is neglected.

In this work the mathematical formulation of the restrictions that really limit the attainable V_{REF} value and make a design of a curvature corrected bandgap reference realizable, are obtained. This value is verified experimentally through the design and measurement of bandgap type circuits, based on the sum of two base-emitter voltages.

Analysis of design equations

The block diagram of a low voltage bandgap reference, based on the sum of two base-emitter voltages is shown in figure 1 [2].

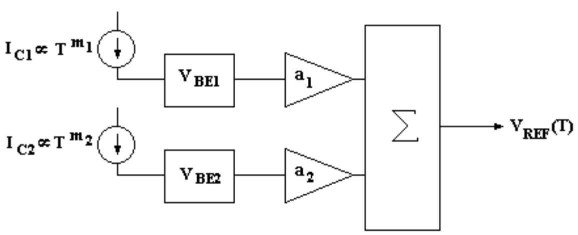

Figure 1 Block diagram of a bandgap reference source with collector current $I_C μT^m$

V_{BE1} and V_{BE2} stand for the expressions of the base-emitter voltages of the transistors and m1 and m2 determine the bias current temperature dependences [2,4,5].

The expression of $V_{REF}(T)$ for the circuit of figure 1 is given by equation (1):

$$V_{REF}(T) = a_1 V_{BE1}(T) + a_2 V_{BE2}(T) \quad (1)$$

Using the linear approximation of bandgap voltage $V_G(T)$ [4,6,7], the expressions of $V_{BE1}(T)$ and $V_{BE2}(T)$ can be written via equations (2) and (3):

$$V_{BE1}(T) = V_{G0}(1-\frac{T}{T_r}) + \frac{T}{T_r}V_{BE1}(T_r) - (\eta - m_1)\frac{k}{q}(T\,ln\frac{T}{T_r}) \quad (2)$$

$$V_{BE2}(T) = V_{G0}(1-\frac{T}{T_r}) + \frac{T}{T_r}V_{BE2}(T_r) - (\eta - m_2)\frac{k}{q}(T\,ln\frac{T}{T_r}) \quad (3)$$

where V_{G0} is the bandgap voltage extrapolated at zero Kelvin and η is a parameter related to the temperature dependence of the mobility of minority carriers in the base region [8]. Their values are obtained from experimental measurements of $V_{BE2}(T_r)$ [4,8].

For the design of bandgap reference the parameters a1, a2, and VBE1(Tr) of expression (1) should be determined, taking into account the known values m_1, m_2 and $V_{BE2}(T_r)$.

In order to reduce the temperature dependence of the reference voltage, that is $V_{REF}(T)=V_{REF,}$ the Taylor criterion will be applied.

Using Taylor criterion at T= T_r, all possible derivates are zero around T_r, so the equations (4), (5) and (6) result:

$$V_{REF}(T_r) = V_{REF} \qquad (4)$$

$$V'_{REF}(T_r) = 0 \qquad (5)$$

$$V''_{REF}(T_r) = 0 \qquad (6)$$

Substituting (2) and (3) in (1), the expression (7) is obtained:

$$\begin{aligned} V_{REF}(T) &= V_{G0}(1-\tfrac{T}{T_r})(a_1+a_2) \\ &+ \tfrac{T}{T_r}[a_1 V_{BE1}(T_r) + a_2 V_{BE2}(T_r)] \\ &- [a_1(\eta - m_1) + a_2(\eta - m_2)]\tfrac{k}{q} T \ln\tfrac{T}{T_r} \end{aligned} \qquad (7)$$

From derivation of (7), equations (8) and (9) result:

$$\begin{aligned} V'_{REF}(T) &= a_1\left[-\frac{V_{G0}}{T_r} + \frac{V_{BE1}(T_r)}{T_r} - (\eta - m_1)\frac{k}{q}\ln\frac{T}{T_r} - (\eta - m_1)\frac{k}{q}\right] \\ &+ a_2\left[-\frac{V_{G0}}{T_r} + \frac{V_{BE2}(T_r)}{T_r} - (\eta - m_2)\frac{k}{q}\ln\frac{T}{T_r} - (\eta - m_2)\frac{k}{q}\right] \end{aligned} \qquad (8)$$

$$V''_{REF}(T) = -[a_1(\eta - m_1) + a_2(\eta - m_2)]\tfrac{k}{q}\tfrac{1}{T} \qquad (9)$$

The use of condition (6) in (9) allows obtaining equation (10):

$$a_1(\eta - m_1) + a_2(\eta - m_2) = 0 \qquad (10)$$

This condition is valid for all the temperature range.

Considering (10) in (7), equation (11) results:

$$\begin{aligned} &V_{REF}(T) \\ &= V_{G0}(1-\tfrac{T}{T_r})(a_1+a_2) + \tfrac{T}{T_r}[a_1 V_{BE1}(T_r) \\ &+ a_2 V_{BE2}(T_r)] \end{aligned} \qquad (11)$$

From where the equation (12) is obtained:

$$a_1 + a_2 = \frac{V_{REF}}{V_{G0}} \qquad (12)$$

Expressions (1), (10) and (12) form an equation system which allows to obtain the parameters a_1, a_2 and $V_{BE1}(T_r)$ as written in (13), (14) and (15):

$$a_1 = \left(\frac{\eta - m_2}{m_1 - m_2}\right)\frac{V_{REF}}{V_{G0}} \qquad (13)$$

$$a_2 = \left(\frac{\eta - m_1}{m_1 - m_2}\right)\frac{V_{REF}}{V_{G0}} \qquad (14)$$

$$V_{BE1}(T_r) = \tfrac{1}{a_1}(V_{REF} - a_2 V_{BE2}(T_r)) \qquad (15)$$

Analysis of the design restrictions

Equations (13) and (14) allow a convenient visualization of the design conditions when they are represented as lines in a1, a2 planes, as shown in figure 2 The solution for the design will be given by the intersection of both lines.

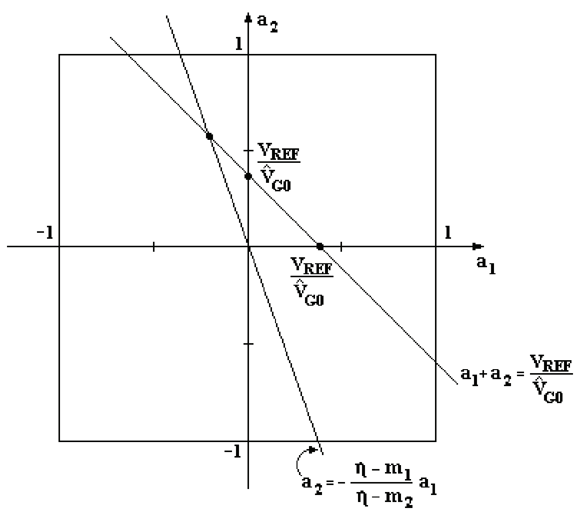

Figure 2 Geometrical representation of equations (13) and (14)

Since only the interval [-1, 1] is admitted for a_1 and a_2 values, the design conditions given by (16), (17) and (18) have to be satisfy:

$$\frac{V_{REF}}{V_{G0}} \langle 1 \Rightarrow V_{REF} \langle V_{G0} \qquad (16)$$

$$\left|\frac{\eta - m_2}{m_1 - m_2}\frac{V_{REF}}{V_{G0}}\right| \langle 1 \Rightarrow \frac{V_{REF}}{V_{G0}} \langle \left|\frac{m_1 - m_2}{\eta - m_2}\right| \quad (17)$$

$$\left|\frac{\eta - m_1}{m_1 - m_2}\frac{V_{REF}}{V_{G0}}\right| \langle 1 \Rightarrow \frac{V_{REF}}{V_{G0}} \langle \left|\frac{m_1 - m_2}{\eta - m_1}\right| \quad (18)$$

For this reason V_{REF} has to be less than the minor of the three equations given by (16), (17) and (18). This means that for a determined design, where m_1 and m_2 values are given by the temperature dependence of the bias currents, and the values of h and V_{G0} depend on the transistor used, V_{REF} can not be greater than certain value that here will be denominated V_{REFMAX}.

Use of the design restrictions in a practical case

Criterion expressed in equations (16), (17) and (18)) was assessed experimentally through the design and measurements of the bandgap type circuit shown in figure 3, using constants (m1=0) and IPTAT (m2=1) bias currents.

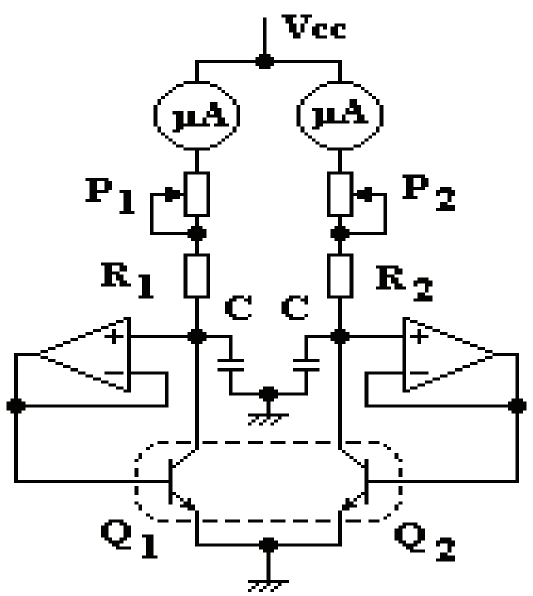

Figure 3 Bandgap type circuit where Q1 and Q2 are MAT01 transistors

A mathematical program was developed as a tool in order to:

Calculate the values of η and V_{G0} for the transistors MAT01 used (based in experimental measurements of V_{BE} at three different temperatures) and V_{REFMAX}, for m_1=0 and m_2=1.

Calculate the design parameters a_1, a_2 and V_{BE1}(Tr) for the wished V_{REF} .

The requirements needed in the measurement of temperature, potential and current to obtain a mistake less than 0.25 ppm/°C in the range 20 to 100 °C were found by means of analysis of sensibility as:

a) Temperature: Resolution better than 0.01 °C in the work range

b) Potential: Resolution better than 1 µV

c) The stability of the potentials V_{BE1}(T) and V_{BE2}(T) better than 0.2 µV respectively

For the measurement the following instruments were used:

- Astandard platinum resistor PT1000 DIN EN60751 as temperature sensor. The resistance of the PT1000 was measured with a Keithley digital electrometer model 614.
- Base-emitter voltages V_{BE1}(T) and V_{BE2}(T) were measured with the Keithley nanovoltmeter model 2182 with 100 nV resolution in a 1 V scale.

For the used MAT01 transistors the following values were obtained: η=5.88818419, V_{G0}=1.08109259 V and V_{REFMAX}=0.18360373 V. These results were used to calculate the design parameters in two cases (The screen shots of the mathematical program are shown in figures 4 and 5 respectively) :

a.) V_{REF} =160 mV < V_{REFMAX} (figure 4)

b.) V_{REF} =200 mV > V_{REFmax} (figure 5)

In screen shot of figure 5 the value of a_1 > 1 can be observed. This value is out of [-1,1] interval (see figure 2) affecting the a_2 value too in equation

(12). , confirming that a design for V_{REF} > 0.18360 V = V_{REFMAX} is not possible.

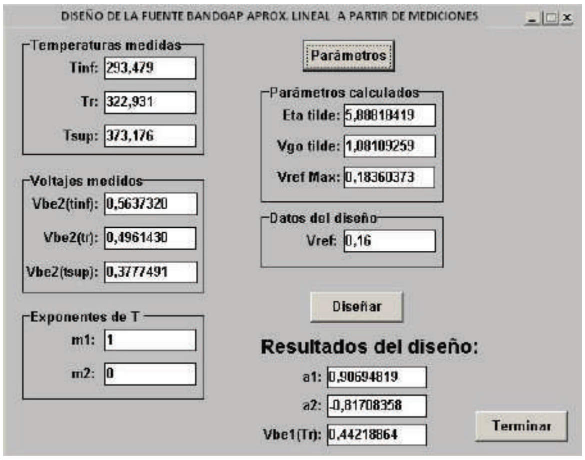

Figure 4 Screen shot of the mathematical program for V_{REF} =160 mV< V_{REFMAX}

The reference voltages were calculated in both cases, using in equation (1) the experimentally measured values of $V_{BE1}(T)$ and $V_{BE2}(T)$ together with the values a_1 and a_2 obtained by the mathematical program. The experimental measurements and calculated V_{REF} values are shown in table 1.

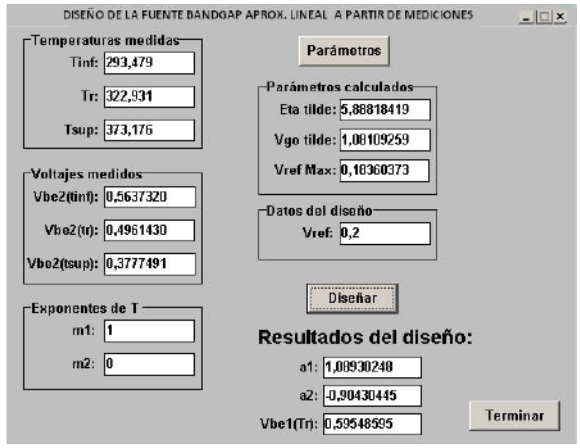

Figure 5 Screen shot of the mathematical program for V_{REF} =200 mV > V_{REFMAX}

Table 1 Experimental data and V_{REF} obtained

Temp. (°C)	*V_{BE2} (V)*	*V_{BE1} (V)*	*V_{REF}< V_{REFMAX} 160 (mV) (a_1=0.90694819 a_2=-0.81708358)*	*V_{REF}> V_{REFMAX} 200 (mV) (a_1=1.08930248 a_2=-0.90430445)*
20.657	0.563238	0.651186	160.00043	130.37998
33.996	0.532382	0.625571	159.99993	132.35925
40.579	0.517263	0.613343	159.99936	133.62281
49.781	0.496143	0.595485	159.99999	134.68400
62.274	0.467055	0.571339	160.00067	136.55175
76.523	0.433315	0.543328	160.00066	138.71659
83.186	0.417628	0.530306	160.00050	139.72294
91.005	0.399010	0.514849	159.99970	140.91671
99.888	0.378010	0.497117	160.00045	141.99338

In table 1 the values of V_{REF} were obtained using equation (1). For V_{REF} > V_{REFMAX} case, V_{REF} values result far away from the design one (200 mV) due to incorrect a_1 and a_2 values, showing that a target design value greater than V_{REFMAX} is not attainable. Besides their increase with temperature invalids its use as a reference.

In figures 6 and 7 the variations of V_{REF} with temperature are shown for both cases.

For $V_{REF} > V_{REFMAX}$ case, V_{REF} values result far away from the design one (200 mV) due to incorrect a_1 and a_2 values.

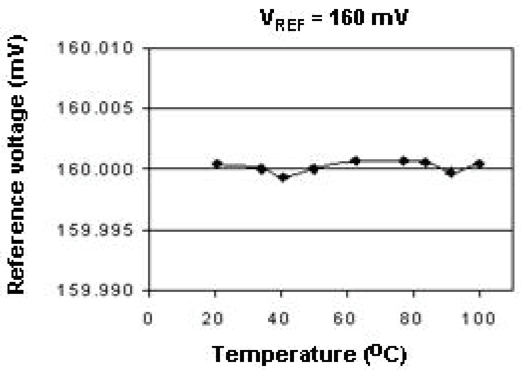

Figure 6 Variation of V_{REF}(T) with temperature for V_{REF} =160 mV < V_{REFMAX}

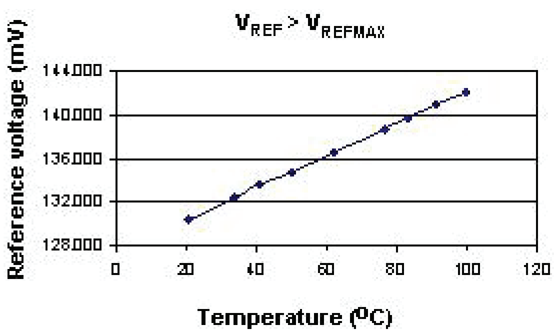

Figure 7 Variation of V_{REF}(T) with temperature for V_{REF} = 200 mV > V_{REFMAX}

The reference voltage in figure 6, designed to 160 mV has an average temperature coefficient of 0.101 ppm/ºC, while the one of the figure 7, designed to 200 mV shows an increase with temperature that invalids its use as a reference.

Conclusions

The analysis and the visualization of the design equations of a bandgap voltage reference based on the sum of two base-emitter voltages lead to obtain three equations that really limit the attainable value of the reference voltage V_{REF}. This means that V_{REF} has to be less than the minor of the three quotas and can not overpass the value of V_{REFMAX}.

This result was assessed experimentally through the design and measurement of a bandgap type circuit. The results show that the reference voltage designed to 160 mV < V_{REFMAX} has an average temperature coefficient of 0.101 ppm/ºC, while that designed to 200 mV > V_{REFMAX} results far away from the design one, increasing with temperature. This behavior invalids its use as a reference. This fact confirms the importance of knowing restrictions that really limit the attainable value of reference voltages, aspect not studied up to now.

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