Decentralized AC power flow for multi-area power systems using a decomposition approach based on Lagrangian relaxation

Flujo de potencia AC descentralizado para sistemas de potencia multi-área usando un método de descomposición basado en relajación Lagrangiana

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Abstract

A decentralized solution method to the AC power flow problem in power systems with interconnected areas is presented. The proposed methodology allows finding the operation point of a particular area without explicit knowledge of network data of adjacent areas, being only necessary to exchange border information related to the interconnection lines between areas. The methodology is based on the decomposition of the first-order optimality conditions of the AC power flow, which is formulated as a nonlinear programming problem. A 9-bus didactic system, the IEEE Three Area RTS-96 and the IEEE 118 bus test systems are used in order to show the operation and effectiveness of the distributed AC power flow.

----- *Keywords*: Decentralized power flow, multi-area power systems, decomposition techniques

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Resumen

Este artículo presenta un método de solución descentralizada para el problema de flujo de potencia AC en sistemas de potencia con áreas interconectadas. La metodología propuesta permite encontrar el punto de operación de un área en particular sin la necesidad de conocer explícitamente los datos de las áreas adyacentes, siendo sólo necesario intercambiar información de frontera relacionada con las líneas de interconexión entre áreas. El método se basa en la descomposición de las condiciones de optimalidad de primer orden del flujo de potencia AC, el cual se formula como un problema de programación no lineal. Un sistema didáctico de 9 barras, el sistema IEEE RTS-96 y el sistema de prueba IEEE de 118 barras se utilizan para mostrar la operación y eficiencia del flujo de potencia AC distribuido.

----- *Palabras clave:* Flujo de potencia descentralizado, sistemas de potencia multi-área, técnicas de descomposición

Introduction

Real power systems are usually composed by interconnected areas controlled by different independent agents known as Regional Operators (ROs). Generally, to obtain the operating point of a particular area, the RO must know all network data, or use an approximate network equivalent for some regions of the system. However, under the new framework of competitive markets, each RO has to face confidentiality and security policies that restrict access to network information. Under this scenario, the concept of parallel computing emerges as a solution that provides flexibility and robustness to the mathematical procedures for coordination of multi-area Electric Power Systems (EPS). The basic idea consists in dividing the overall problem into several sub-regional problems associated with each area, where each RO solves its own optimization problem. Subsequently, an iterative process of global coordination exchanges strategic information between areas. The solution is the same that would be reached with a traditional centralized power flow. The main reasons for adopting a decentralized power flow framework are:

- Full knowledge of network or market data of adjacent areas is not required. Form the point of view of any RO, adjacent areas can be seen as "black boxes". The only data required is related with border information of the lines interconnecting different areas.
- The proposed multi-area approach constitutes an operational and mathematically robust methodology. If there is a difficulty in solving a regional problem or a failure in communication it is still possible to solve the other sub-problems.
- The algorithm is designed to be implemented using parallel processing techniques. Each RO is a processor that solves its own regional problem.
- A Distributed Power Flow (DPF) tool can be seen as a basic building block able to address a large number of problems under a multi-area competitive market philosophy.

There are several reports addressing the concept of multi-area power flow (PF) and optimal power flow (OPF) in the specialized literature. Some of the methodologies regarding multi-area OPF require the solution of a global PF, when this happens; the concept of parallel computing is lost. For instance, [1] provides a security-constraint multi-area OPF using the Auxiliary Problem Principle (APP). However, the method is not fully distributed because it requires a global PF in each iteration of the coordination algorithm in order to identify possible congested

lines and various factors associated with post-contingency constraints. In [2] a nonlinear DC model is used to solve the OPF problem. The algorithm is based on the Lagrangian relaxation decomposition procedure. In [3] a DC model is also used and the Decomposition of First-order Optimality Conditions (DFOC) method presented in [4] and [5] is used. In a later paper the same authors presented an extension of their work using an AC model and a composite objective function [6]. In [7] and [8], Kim et al. propose an approach that is suitable for distributed implementation using the APP method and a linearized augmented Lagrangian approach to improve convergence. In [9] two OPF decomposition methods are compared: one based on an adjustment procedure at a common interface, and the other based on the passing of adjacent variables. The performance of the last one is enhanced by introducing a master-slave principle. In [10] a decomposition method considering overlapping areas is presented. In this case, the authors include the operation and control of FACTS devices in the modeling.

Regarding to methodologies that aim to solve an AC-PF in a distributed or decentralized fashion, it is possible to classify the solution techniques in literature into two groups: i) using network equivalents and ii) using mathematical decomposition techniques. Associated with the first group, in [11] a distributed algorithm for security static analysis of a multi-area system is proposed. A reference node is defined for each area of the system, and consequently, it is necessary to use a synchronization process of angles between the reference nodes in different areas. An assessment of different network equivalent types for the implementation of a decentralized control scheme for multi-area systems is presented in [12]. The main drawback when using a network equivalent is that the optimization process performance is highly dependent on the type of equivalent chosen. Additionally, various network equivalent types can be found for one system, and different adjustment procedures are necessary to reduce the errors introduced by the approximations of each equivalent.

In what regards mathematical decomposition techniques, few references are found in which the PF problem is solved (most of them are applied to the OPF problem instead). In [13] A decentralized DC-PF using the APP is presented. The goal is to preserve network data confidentiality for each RO in an open and competitive electricity market.

The algorithm presented in this paper is an application of the Lagrangean decomposition method proposed in [5]. The AC-PF is modeled as a non-linear programming problem, so that the equivalent model can then be physically decoupled around the tie- lines. Subsequently, a regional model using DFOC is obtained and the Newton system is built to be solved iteratively at each regional problem. The goal is not computational-efficiency improvement of the PF. Most significant in this paper is to provide a tool to solve the multi-area AC-PF maintaining an independent operation of each RO to achieve the same operating point found by a centralized AC-PF. With regard to the decentralized power flow model using DFOC as proposed in this paper, no reference was found in the literature.

Results show that the system operating point obtained by the centralized PF, is the same as that obtained by the DPF, maintaining the independent operation of each area. The centralized scheme generally presents better computational times than the one obtained by the distributed scheme. However, it is worth to mention that the main focus of this paper is not to reduce computational time, but to provide an approach for a DPF. Nevertheless, to reduce the computational time of the DPF, the following actions are recommended: i) Using decoupling techniques ii) Implementing parallel processing techniques, in which each OR is a processor. iii) Incorporating strategies that approximate search directions without the need of reaching the optimality of each regional problem. iv) Developing coordination strategies to accelerate the voltage angle convergence.

Feasible solution of an AC-PF

The AC-FP formulation presented in this paper uses the polar representation described in [14].

For a system of N nodes there will be N nodal equations given by (1).

$$V_k e^{-j\theta_k} \sum_{m=1}^{N} \left(G_{km} + jB_{km}\right) V_m e^{j\theta_m} = P_{N_k} - jQ_{N_k}; \quad k = 1,2,\ldots,N \tag{1}$$

Where V_k and θ_k are the voltage magnitude and angle respectively. $G_{km} + jBk_m$ is the nodal admittance matrix element related to nodes k and m. P_{N_k} and Q_{N_k} are the net active and reactive power injected in node k respectively. Defining the active and reactive power injections as functions of the voltage magnitude and angle as shown in (2), equation (1) can be divided into $2N$ real nodal equations as shown in (3) and (4).

$$V_k e^{-j\theta_k} \sum_{m=1}^{N} \left(G_{km} + jB_{km}\right) V_m e^{j\theta_m} = P_k(V,\theta) - jQ_k(V,\theta) \tag{2}$$

$$P_k(V,\theta) - P_{N_k} = 0 \qquad k = 1,2,\ldots,N \tag{3}$$

$$Q_k(V,\theta) - Q_{N_k} = 0 \qquad k = 1,2,\ldots,N \tag{4}$$

Equations (3) and (4) represent the power balance on a typical node. Defining X as the unspecified variables vector ($X = [V,\theta]^T$), the system of equations (3)-(4) can be represented as shown in (5).

$$\left[g(X)\right] = \begin{bmatrix} Equation\ (3) \\ Equation\ (4) \end{bmatrix} For\ each\ node\ PQ \\ Equation\ (3)\ For\ each\ node\ PV \tag{5}$$

Where $[g(X)]$ represents a vector of size ndg equal to the number of equality constraints. Applying the first-order Taylor series expansion to (5), the set of linear equations given by (6) is obtained. The Newton method consists on the successive solution of equation (6), updating in each iteration the state variables as shown in (7).

$$\left[\left.\frac{\partial g(X)}{\partial X}\right|_{X=X^{(h)}}\right] \cdot [\Delta X] = -\left[g\left(X^{(h)}\right)\right] \tag{6}$$

$$\left[X^{(h+1)}\right] = \left[X^{(h)}\right] + [\Delta X] \tag{7}$$

In (6) the term $[\partial g/\partial X]$ is known as the Jacobian matrix, denoted by $\lfloor J_g \rfloor$. The vector $[g(X^{(h)})]$ corresponds to the power errors. These errors are given by equations (8), (9) and (10). Where $\Delta S = [\Delta P\ \Delta Q]$ represents the vector of apparent power errors. Note that in this case the super index T stand for the transposed of the vector.

$$\left[\Delta S\left(X^{(h)}\right)\right] = -\left[g\left(X^{(h)}\right)\right] = [\Delta P, \Delta Q]^T \tag{8}$$

$$[\Delta P] = [P_N] - \left[P\left(V^{(h)}, \theta^{(h)}\right)\right] \tag{9}$$

$$[\Delta Q] = [Q_N] - \left[Q\left(V^{(h)}, \theta^{(h)}\right)\right] \tag{10}$$

Finally, equation (6) can be rewritten as shown in (11).

$$\left[\left.\frac{\partial g(X)}{\partial X}\right|_{X=X^{(h)}}\right]^T \cdot [\Delta X] = \left[\Delta S\left(X^{(h)}\right)\right] \tag{11}$$

AC-PF as a nonlinear programming problem

The AC-PF described in the previous section can be formulated as a nonlinear programming problem (NLP), where the objective function consists in minimizing the sum of the squared power errors as shown in (12).

$$Min \quad f(X) = \frac{1}{2}\left([g(X)]^T \cdot [g(X)]\right) \quad s.t: \quad [g(X)] = 0 \tag{12}$$

The solution to this problem is subject to compliance with the nodal power equations (3) and (4). The unrestricted Langrangean function is presented in (13). Where $[\lambda]$ is known as the Lagrange multiplier vector.

$$L(X,\lambda) = f(X) + [\lambda]^T [g(X)] \quad (13)$$

Applying the first order optimality conditions and solving by Newton method the system of nonlinear equations given by (14) is obtained. Where the Hessian matriz [*H*] and the vector of independent parameters [*r*] are given by (15) and (16) respectively.

$$\begin{bmatrix} [H] & [J_g]^T_{X=X^{(h)}} \\ \hline [J_g]_{X=X^{(h)}} & [0] \end{bmatrix} \quad (14)$$

$$\begin{bmatrix} [\Delta X] \\ \hline [\Delta\lambda] \end{bmatrix} = \begin{bmatrix} [r] \\ \hline [g(X)]_{X=X^{(h)}} \end{bmatrix}$$

$$[H] = [J_g] + \sum_{j=1}^{ndg} \lambda_j^{(h)} \left[H_{g_j} \right] \Big|_{X=X^{(h)}} \quad (15)$$

$$[r] = \left[g\left(X^{(h)}\right) \right] + [J_g]^T_{X=X^{(h)}} [\lambda] \quad (16)$$

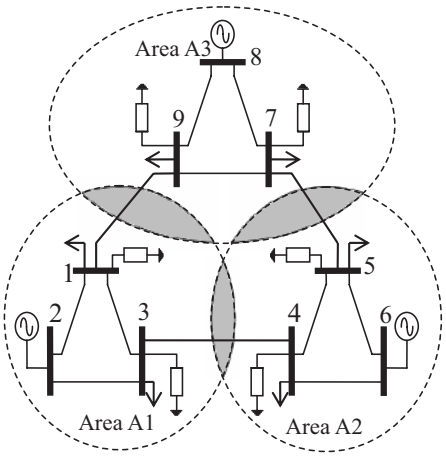
Where $\lfloor H_{gj} \rfloor$ is the Hessian matrix of equality constraint *j*. Note that it is not necessary to calculate the Lagrange multipliers to solve the AC-PF. Consequently, to obtain the search directions ΔX from (14), it is only necessary to solve the system of equations given by (17).

$$[J_g]_{X=X^{(h)}} [\Delta X] = -\left[g\left(X^{(h)}\right) \right]_{X=X^{(h)}} \quad (17)$$

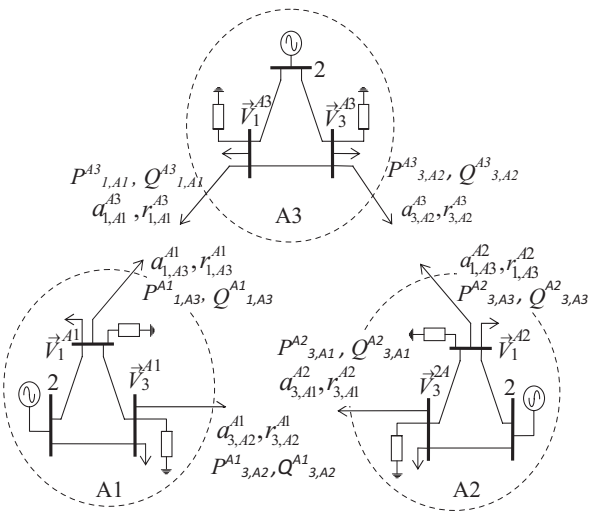
This system allows for a feasible solution of the AC-PF problem equal to that obtained in (6). In the DPF scheme the Lagrange multipliers become more important and should be calculated. These multipliers constitute important additional information associated with import and export costs of active and reactive power between the interconnected areas which can be used in electricity market applications.

Decomposition scheme

A necessary condition for a trivial decomposition of a NLP problem is that both, the objective function and the constraints are separable. That is, the variables of the NLP problem must be associated with only one of the sub-problems resulting from the decomposition process. In order to achieve a trivial decomposition of the EPS, a separation around the elements that connect the different areas (tie-lines) is implemented as illustrated in figure 1.



(a) Three-area centralized system



(b) Duplication of boundary variables

Figure 1 EPS Decomposition scheme

The decomposition is performed using fictitious generators representing flows through the tie-lines. Four new variables per tie-line (border variables) are added to the original problem, as shown in figure 1b. These border variables

represent the necessary fictitious generation for the feasibility of the problem in area A. In general terms, this fictitious generation will be denoted as $S_F^A = \left[P_F^A, Q_F^A\right]^T$. If there is generation in the border bus, this bus is classified as PV, otherwise, as bus PQ. The equivalent NLP problem is described by equations (18), (19), (20) and (21) as shown bellow.

$$Min \qquad \sum_A (f(X^A)) \tag{18}$$

$$s.t. \quad \left[g(X^A)\right] = 0 \tag{19}$$

$$\left[P_F^A\right] - \left[P_{tie-line}\left(X^A, X^{\mathcal{A}}\right)\right] = 0 \tag{20}$$

$$\left[Q_F^A\right] - \left[Q_{tie-line}\left(X^A, X^{\mathcal{A}}\right)\right] = 0 \tag{21}$$

Where $P_{tie\text{-}line}$ and $Q_{tie\text{-}line}$ are vectors of calculated powers (active and reactive), that flow between interconnected areas. These powers are calculated, based on the current state variables values in the border buses of each area ($X^A = [V^A, \theta^A]^T$, $X^{AA}, = [V^{AA}, \theta^{AA}]^T$).

The constraints (20) and (21), known as coupling constraints, are necessary to coordinate the overall optimization process, and ensure that at the optimal point, problem (12) is equivalent to problem (18)-(21). Due to the fact that the coupling constraints are function of variables X^{AA}, which belong to adjacent areas, the problem (18)-(21) cannot be separated in a direct or trivial way, being necessary to use a mathematic decomposition method.

Regional problem

The DFOC method is based on the decomposition of the optimality conditions of the original global problem given by (18)-(21). This decomposition is performed in such a way that the linear combination of the Karush-Kuhn-Tucker (KKT) optimality conditions of all regional problems in the k-th iteration of a global coordination process is identical to the KKT conditions of the original problem at the optimum point. In this iterative process, the values of the variables corresponding to the border bus from an adjacent area $\left(\overline{X}^{AA} = \left[\overline{V}^{AA}, \overline{\theta}^{AA}\right]\right)$ are known from the previous iteration. Therefore, the NLP regional problem for the area A is given by (22).

$$\begin{aligned} \underset{X^A, S_F^A}{Min} \quad & f(X^A) + \left[\overline{u}^{AA}\right]^T \left[S_F^A\right] \\ s.t. \quad & \left[g(X^A)\right] = 0 \\ & \left[g_F(X^A, S_F^A)\right] = 0 \end{aligned} \tag{22}$$

Where f is the regional problem objective function. The vector $[g_F]$ represents the constraints sets (20) and (21) associated with the fictitious generation in the border buses. The second term of the objective function corresponds to the import costs of active and reactive power, which can be rewritten as in (23).

$$\begin{aligned} & \left[\overline{u}^{AA}\right]^T \left[S_F^A\right] = \left[\overline{a}_{tie-line}^{AA}\right]^T \\ & \left[P_{tie-line}\left(X^A, \overline{X}^{AA}\right)\right] + \\ & \left[\overline{r}_{tie-line}^{AA}\right]^T \left[Q_{tie-line}\left(X^A, \overline{X}^{AA}\right)\right] \end{aligned} \tag{23}$$

The Lagrange multipliers of the coupling constraints $\left(a_{tie-line}^A\right)$ and $\left(r_{tie-line}^A\right)$ are the active and reactive power export costs from area A to other adjacent areas $AA\left(\left[u^A\right] = \left[a_{tie-line}^A, r_{tie-line}^A\right]^T\right)$. Therefore the objective function of problem (22), aims to minimize the cost of importing active and reactive power from other areas. These multipliers are used by other areas as import costs for active and reactive power, in which case they are known from the previous iteration and denoted as $\overline{a}_{tie-line}^{\mathcal{A}}$ and $\overline{r}_{tie-line}^{\mathcal{A}}$, respectively. To calculate the power flow from bus k, located in area A, to bus m, located in the adjacent area AA, equations (24) and (25) are used. Where B_{sh} is the tie-line shunt susceptance.

$$P_{tie-line} = \left(V_k^A\right)^2 G_{km} - V_k^A V_m^{AA} G_{km} \cos\left(\theta_k^A - \theta_m^{AA}\right) - V_k^A \overline{V}_m^{AA} B_{km} \sin\left(\theta_k^A - \overline{\theta}_m^{AA}\right) \quad (24)$$

$$Q_{tie-line} = -\left(V_k^A\right)^2 \left(B_{km} + B_{sh}\right) + V_k^A \overline{V}_m^{AA} B_{km} \cos\left(\theta_k^A - \overline{\theta}_m^{AA}\right) - V_k^A \overline{V}_m^{AA} G_{km} \sin\left(\theta_k^A - \overline{\theta}_m^{AA}\right) \quad (25)$$

Convergence criterion

The decentralized algorithm terminates when equations (26) and (27) are satisfied per tie-line of the overall system. For all KKT points, the equations must be satisfied with a tolerance ε_P (MW) and ε_Q (MVAR). These equations are the result of analyzing the active and reactive power flows on each tie-line, taking into account the power losses in the tie-line.

$$\Delta P_{km} = \left|\overline{P}_{F_k}^A + \overline{P}_{F_m}^{AA} - P_{loss}\left(\overline{V}_k^A, \overline{V}_m^{AA}, \overline{\theta}_k^A, \overline{\theta}_k^{AA}\right)\right| \le \varepsilon_P \quad 26)$$

$$\Delta Q_{km} = \left|\overline{Q}_{F_k}^A + \overline{Q}_{F_m}^{AA} - Q_{loss}\left(\overline{V}_k^A, \overline{V}_m^{AA}, \overline{\theta}_k^A, \overline{\theta}_k^{AA}\right)\right| \le \varepsilon_Q \quad (27)$$

$\overline{P}_{F_k}^A, \overline{P}_{F_m}^{AA}, \overline{Q}_{F_k}^A, \overline{Q}_{F_m}^{AA}, \overline{V}_k^A, \overline{V}_m^{AA}, \overline{\theta}_k^A, \overline{\theta}_k^{AA}$ are known values obtained from the decentralized optimal solution of problem (22). An area Aconverges when the errors ΔP_{km} and ΔQ_{km}, of all its tie-lines ($k \in A\, y\, m \in AA$), are within specified tolerances. Once an area A has converged, it is not necessary to solve the regional problem associated with this area in the next iteration of the global coordination process.

Global coordination algorithm

The global coordination algorithm of the proposed multi-area AC-PF is described below:

Step 1. Define initial conditions in all areas: the initial values of the Lagrange multipliers as well as the fictitious generation in all areas can be zero.

Step 2. Solve problem (22) for the areas that have not converged. Solving this problem the state variables vector and the Lagrange multipliers are obtained. This process can be carried out using parallel computing techniques.

Step 3. Check convergence criterion evaluating equations (26) and (27). If the convergence criterion is satisfied by all areas, then the process ends; otherwise go to step 4.

Step 4. Exchange border variable information and Lagrange multipliers among all areas and go to step 2.

Newton system

The Newton system obtained from the optimization problem described by (22) after applying the same process done with problem (12) is presented in (28) and (29).

$$\begin{bmatrix} [H^A] & 0 & [J_g]^T_{X^A=X^{A(h)}} & [J_{gF}]^T_{\substack{X^A=X^{A(h)} \\ S_F^A=S_F^{A(h)}}} \\ 0 & 0 & 0 & [I] \\ [J_g]_{X^A=X^{A(h)}} & 0 & 0 & 0 \\ [J_{gF}]_{\substack{X^A=X^{A(h)} \\ S_F^A=S_F^{A(h)}}} & [I] & 0 & 0 \end{bmatrix} \begin{bmatrix} [\Delta X^A] \\ [\Delta S_F^A] \\ [\Delta \lambda^A] \\ [\Delta \mu^A] \end{bmatrix} = \quad (28)$$

$$\begin{bmatrix} \left[\frac{\partial \ell}{\partial X^A}\right]_{X^A=X^{A(h)}} & \left[\frac{\partial \ell}{\partial S_F^A}\right]_{\substack{X^A=X^{A(h)} \\ S_F^A=S_F^{A(h)}}} & \left[g(X^A)\right]_{X^A=X^{A(h)}} & \left[g(X^A, S_F^A)\right]_{\substack{X^A=X^{A(h)} \\ S_F^A=S_F^{A(h)}}} \end{bmatrix}^T$$

$$[H^A]=[J_g]+\sum_{j=1}^{ndg}\lambda_j^{(h)}[H_{g_j}]_{X^{A(k)}}+\sum_{j=1}^{ndf}\mu_j^{(h)}[H_{g_{F_j}}]_{\substack{X^{A(k)}\\ S_F^{A(k)}}} \quad (29)$$

Where $J_g=\partial g(X^A)/\partial X^A$, $J_{g_F}=\partial g_F(X^A,S_F^A)/\partial X^A$. H_{gj} and H_{gFj} are Hessian matrices of each equality constraint. The major computational effort is solving the symmetric Newton system shown in (28). Since it is a sparse system, the sparse techniques presented in [15] and [16] can be used to save memory space and improve speed.

Results

This section presents the results obtained with three test systems: a 9 bus system, like the one shown in figure 1, the IEEE Three Area RTS96 and the IEEE 118 bus test system.

9-Bus test system

The centralized bus and branch data of this system can be consulted in [17]. Table 1 shows the state variables obtained using the centralized AC-PF, with a tolerance of $1x10^{-6}$ p.u. This system operation point produces the power flows in the interconnection lines shown in table 2. Table 3 shows the tie-line power errors and the convergence of each area. In this case the number 1 indicates the iteration in which an area has converged. The process ends when all areas meet the convergence criteria, which occurs in iteration 44.

In the last three columns of table 3 it can be noted that the area *A*2 satisfies the convergence criterion repeated and successive times during the iterative process. When one or more areas converge at iteration *k*, it is not necessary to solve the regional problem for these areas during the next iteration.

It was observed that the algorithm early reaches final values for the voltage magnitude (from iteration 21). Moreover, the nodal voltage angles convergence between all system areas requires a larger number of iterations because of the angle coordination with respect to the reference bus from area *A*1. The behavior of the errors ΔP_{km} and ΔQ_{km} in each tie-line is shown in figure 2. Tie-lines 3-4 and 5-7 show similar behavior and reach acceptable errors levels of active and reactive power from iteration 21. These two tie-lines define the convergence of area *A*2, therefore the regional problem of this area is rarely executed after iteration 21. Tie-line 1-9 is part of the tie-line set that define the convergence of areas *A*1 and *A*3. This tie-line only reaches values within the tolerance at iteration 44. Using a tolerance equal to $1x10^{-6}$ p.u the DPF problem converges to the same operation point as the one shown in tables 3 and 4 after 106 iterations.

Table 1 State variables of the 9-bus test system (centralized approach)

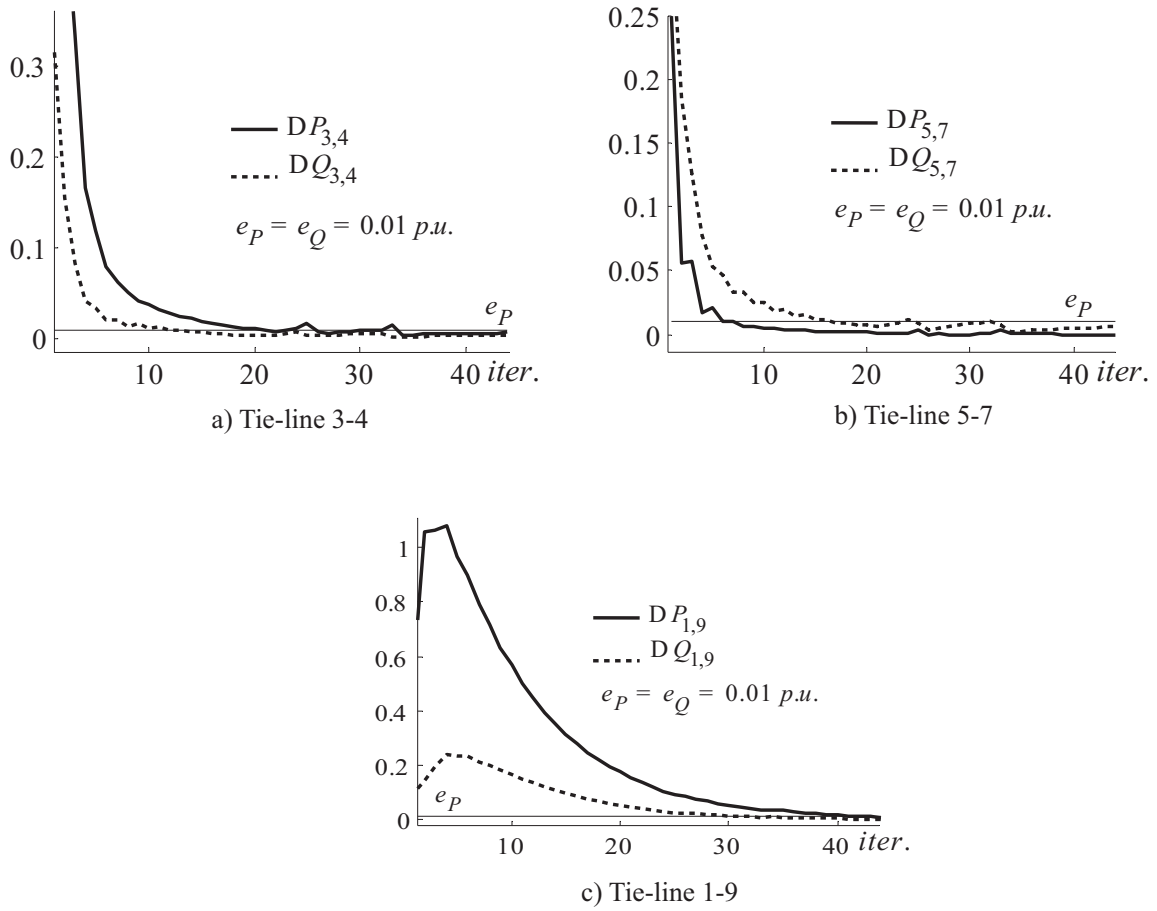
Bus	*V*	*Angle*	P_G	Q_G
1	0.9742	-8.9568	0.0000	0.0000
2	1.0500	0.0000	3.2658	0.4280
3	0.9583	-12.5240	0.0000	0.0000
4	0.9256	-29.4058	0.0000	0.0000
5	0.9539	-26.9135	0.0000	0.0000
6	1.0500	-26.7382	1.0000	1.5041
7	0.9639	-12.1999	0.0000	0.0000
8	1.0500	-2.9814	2.5000	0.5426
9	0.9751	-9.5020	0.0000	0.0000

Table 2 Tie-line power flows and losses of the 9-bus test system (centralized approach)

Tie-line	P_{ij}	Q_{ij}	P_{ij}	Q_{ij}	P_{loss}	Q_{loss}
3-4	1.1695	-0.2370	-1.0144	0.5471	0.1550	0.3101
5-7	-0.0686	0.4504	0.1826	-0.4225	0.1140	0.0278
1-9	0.3388	-0.1763	-0.3374	0.1796	0.0014	0.0034

Table 3 Tie-line power errors and overall convergence

Iter	$DP_{3,4}$	$DQ_{3,4}$	$DP_{5,7}$	$DQ_{5,7}$	*A1*	*A2*	*A3*
1	1.245200	0.315520	0.055578	0.184520	0	0	0
2	0.525770	0.154900	0.055578	0.184520	0	0	0
3	0.334650	0.083274	0.056693	0.125140	0	0	0
…							
21	0.009584	0.003472	0.001269	0.005977	0	1	0
22	0.007043	0.004570	0.000216	0.007099	0	1	0
23	0.008966	0.006072	0.000502	0.009558	0	1	0
…							
42	0.006294	0.003704	0.000034	0.005323	0	1	0
43	0.006454	0.003825	0.000001	0.005561	0	1	0
44	0.006602	0.003921	0.000020	0.005766	1	1	1



a) Tie-line 3-4

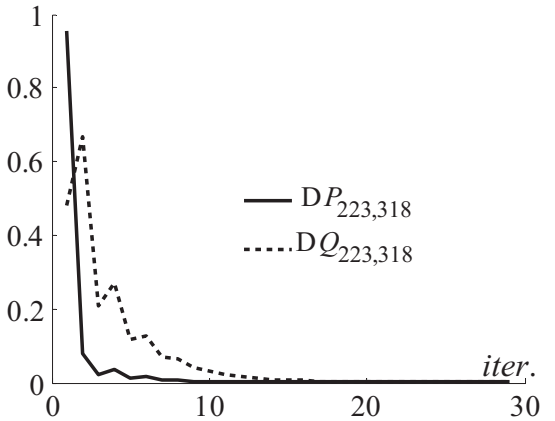
b) Tie-line 5-7

c) Tie-line 1-9

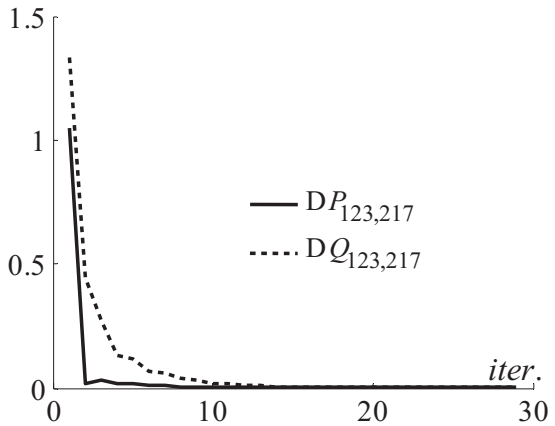
Figure 2 Errors ΔP_{km} and ΔQ_{km} in the tie-lines of the 9-bus test system

IEEE Three Area RTS-96 system

The second power system in which the proposed methodology was tested is the IEEE Three Area RTS-96 system. A full description of the configuration of this system, as well as branch, and bus data can be consulted in [18]. The DPF was run using a tolerance of $\varepsilon_p = \varepsilon_p = 0.0001$p.u. The problem converged after 29 iterations with the same accuracy as that obtained by a traditional centralized PF. The behavior of the errors in tie-lines 223-318 and 123-217 during the iterative process is shown in figure 3. The convergence decreasing property of power errors is similar in all tie-lines of all systems studied, which shows good convergence characteristics of the DPF.



a) Tie-line 223-318

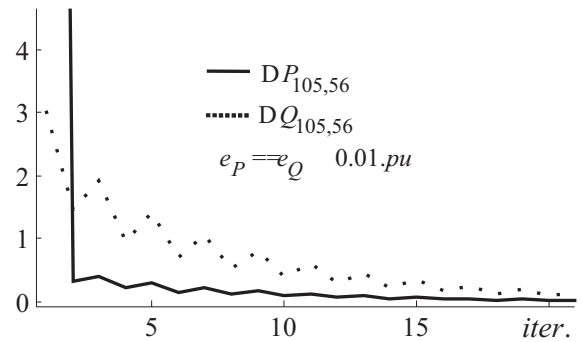


b) Tie-line 123-217

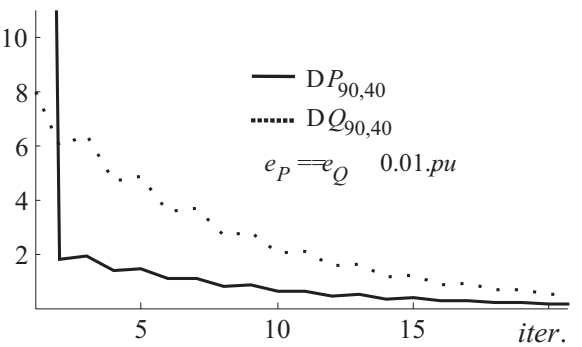
Figure 3 ΔP_{km} and ΔQ_{km} errors in the IEEE RTS-96 tie-lines

IEEE 118 bus test system

The proposed methodology was also test with the IEEE 118 bus test system with two identical areas forming an equivalent system of 236 buses and 374 branches. The areas were connected with tie-lines in buses 105-56 and 90-40 respectively. The DPF was run using a tolerance of $\varepsilon_p = \varepsilon_p = 0.01$ p.u. The problem converged after 18 iterations to the same operation point as obtained by a traditional centralized PF. The behavior of the errors in the tie-lines is presented in figure 4. It can be observed that for both tine-lines the convergence criteria is first met by the active power.



a) Tie-line 105-56



b) Tie-line 90-40

Figure 4 ΔP_{km} and ΔQ_{km} errors in the tie-lines of the two-area IEEE 118 bus test system

Conclusions

A decentralized AC power flow approach for multi-area EPS was presented in this paper. The proposed methodology preserves confidentiality

of network data in each area, allowing an independent operation of each RO. Only border information of each area is exchanged. Also, the proposed AC-PF methodology constitutes a basic tool that can be used in different studies regarding planning and operation of EPS. In all test systems the DPF converged successfully, and the results reached the same level of accuracy as those obtained using a centralized AC-PF. The decomposition method used was shown to be mathematically robust and suitable for large systems. Future work will include the application of this methodology in voltage stability studies and the development of a decentralized probabilistic load flow among others.

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