Ricatti stochastic filter as an estimator

Filtro estocástico de Ricatti como estimador

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Abstract

The Ricatti stochastic digital filter as an estimator is based on a first differences order model with uncorrelated innovation properties bounding the spatial operation region with two auxiliary equations. This permits optimal results having the traditional inversion instead of the pseudo-inverse strategy. The stationary conditions and uncorrelated trajectories were the tools applied in adaptive estimation and identification integrated form. In spite of the black-box form observing the output system, the parametre and identification simulation achieved a great convergence rate in agreement with functional error. It was built considering the identification second probability moment defined as the difference between the desired signal and the output filter response. The parametres estimated were inside the unit circle and had a great advantage because the primitive solution depends on their values.

---------- *Keywords:* Instrumental variable, least squares method, second probability moment, Ricatti estimation, convergence

Resumen

El filtro estocástico de Ricatti como un estimador está basado en un modelo en diferencias de segundo grado, de primer orden con el proceso de innovación no correlacionado acotado por la región del espacio de operaciones a través de dos ecuaciones auxiliares que permiten tener resultados óptimos con la inversión tradicional en vez de usar la estrategia de la pseudo-inversa.

Las trayectorias no correlacionadas y las condiciones estacionarias fueron las herramientas aplicadas en la forma integrada adaptable del estimador con el identificador. A pesar de todas las restricciones del sistema de caja negra en la

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recursividad, fue simulado con una gran relación de convergencia observada en el funcional. Este fue construido considerando el segundo momento de probabilidad del erro de identificación definido por la diferencia entre la señal deseada y la respuesta del filtro. Los parámetros estimados se encuentran dentro del círculo unitario y representa una gran ventaja para el sistema en diferencias ya que su primitiva depende de estos valores.

---------- *Palabras clave:* Variable instrumental, mínimos cuadrados, segundo momento de probabilidad, estimador de Ricatti, convergencia

Introduction

Adaptive Control Theory (ACT) in Digital Systems (DS) uses some mechanisms adjusting the gains into a controlled system according to a reference model answer. These mechanisms are known as Digital Filter Estimation (DFE) techniques and affect the control actions before the feedback ends, adapting its parametres set to a reference objective. For example, the discrete Proportional Integral and the Derivative (PID) control has three gains (K_p, K_i, K_d) and commonly use the Butterworth method in the calculus coefficients, without changes through the time never considering if the system has or has not variations. Without other options, this control is applied in real conditions adjusted experimentally, against the natural evolution system conditions.

The control law action in a digital system operates in finite differences where the order is represented by the output delays signal. The parametre filter estimation considers the output stage order as an element on its dimension [1], using some of the following tools: *Least Square Method* (LSM), *Instrumental Variable* (IV), *Forgetting Factor* (FF), or *Kalman Filter* (KF). In the black-box system scheme, the unobservable internal states and unknown parametres have one of the following conditions:

a) Stationary. The input-output signals rate is smooth or rigid [2] needing some tools in estimation techniques:

a_1) Pseudo-inverse traditional form [3] applied LSM, IV, and FF techniques,

a_2) Correlation inverse matrix used as a Kalman filter gain [4],

a_3) Second probability moment and gradient based on linear stochastic model $Ay_k + \xi_k = u_k$, with uncertainty around the equilibrium solution [5], and $y_k, \xi_k, u_k \in \left(\Re_{[0,1]}, \Im(\Re), P_{[0,1)} \right)$, with $\{\xi_k\} \subseteq N(\mu_\xi, \sigma_\xi^2 < \infty)$, $\{y_k\} \subseteq N(\mu_y, \sigma_y^2 < \infty)$, and $\{u_k\} \subseteq N(\mu_u, \sigma_u^2 < \infty)$ has in common with black-box scheme, the unknown internal matrix parametres A,

a_4) Output natural frequency system bounded all on-line techniques [6 - 8].

a_5) Mean square error applied on adaptive filter convergence without losing its stability [9, 10].

The simulation results, in all cases showed that the functional error converges exponentially to an equilibrium point.

b) Non-stationary. The input-output signals rate has a different velocity set in each time interval, needing some tools in estimation techniques:

b_1) Output matrix variances [11] applied into adaptive filter with Maximum Likelihood,

b_2) Maximal output amplitude [12] into outlier M-estimator,

b_3) Input-output covariance weights [13] used in quasi-maximum likelihood.

The input and output signals had high changes with marginal stable conditions; but in spite of, the estimations converge in a good distribution sense.

The filter estimation is used in:

1) Electrical rotor-position requiring specific velocity using a filter technique affecting the control law amplitude voltages.

2) Multi-Input Multi-Output Orthogonal Frequencies-Division and Multi-Access (MIMO-OFDMA) system had offset up-link non-stationary frequencies [6] dynamically adjusting their LSM gains [14, 15]. The results were limited to smooth conditions.

3) The Signal-to-Noise Ratio (SNR) is seen as a system answer having non-stationary conditions. The estimation results had good distribution convergences [7, 16]. The azimuth and elevation variations in short-range and low-flying air routes were predicted with the LSM Ricatti parametres estimation method [17].

4) The 3D affine motion with two cameras via observations as a single feature point estimates the nine rotational, three translational and 3D position parametres using the LSM Ricatti estimation method [18].

Different estimation techniques were resumed observing the output system with or without stationary conditions. The present paper considers the Auto Regressive Moving Average model ARMA (1, 1) with two auxiliary equations solving the vector estimation using the traditional inverse technique instead of the pseudo-inverse adjoin solution.

Development

The Single-Input Single-Output (SISO) black-box system in the Ricatti differences (1).

$$Y_k = a_k U_k + b_k \Psi_k + c_k \Theta_k \qquad (1)$$

Where $Y_k = E\{y_i\} \in \Re$, $U_k = kE\{u(k)\} \in \Re$ $\Psi_k = E\{\psi_i\} \in \Re$, $\Theta_k = E\{\psi_i^2\} \in \Re$ and $u(k)$ as an unitary excitation, $U_k = const$. Therefore the Ricatti form (1) has a complete solution using two auxiliary equations [19].

Theorem 1. The system described in (1) considering two auxiliary equations (2) and (3).

$$\Gamma_{1,k} = a_k \Psi_k + b_k \Theta_k + c_k \Lambda_k \qquad (2)$$

$$\Gamma_{2,k} = a_k \Theta_k + b_k \Lambda_k + c_k \Phi_k \qquad (3)$$

With its elements defined as: $\Gamma_{1,k} = E\{v_i \psi_i\}$, $\Psi_k = E\{\psi_i\}$, $\Theta_k = E\{\psi_i^2\}$ $\Lambda_k = E\{\psi_i^3\}$ $\Gamma_{2,k} = E\{v_i \psi_i^2\}$ $\Phi_k = E\{\psi_i^4\}$, where the recursive parametres is described from (4) to (6).

$$\hat{a}_k = \beta_k \hat{a}_{k-1} + D_k - E_k - F_k + G_k - H_k + I_k + J_k \qquad (4)$$

$$\hat{b}_k = \delta_k \hat{b}_{k-1} + L_K + M_K - N_K \cdot O_K + P_K' + Q_K' - R_K' + S_K' - T_K' \cdot U_K' \qquad (5)$$

$$\hat{c}_k = \varphi_k \hat{c}_{k-1} + D_K'' + E_K'' - F_K'' - G_K'' + H_K'' - I_K'' - J_K'' + L_K'' + M_K'' \cdot N_K'' \qquad (6)$$

Proof: In a matrix sense, according to (1), (2) and (3), the matrix solution is described in (7).

$$[M_k] = [A_k]^{-1} [Y_k] \qquad (7)$$

In (7) its elements have the form (8).

$$[M_k]^T := [a_k \quad b_k \quad c_k],$$

$$A_k := \begin{vmatrix} k & E\{\psi_i\} & E\{\psi_i^2\} \\ E\{\psi_i\} & E\{\psi_i^2\} & E\{\psi_i^3\} \\ E\{\psi_i^2\} & E\{\psi_i^3\} & E\{\psi_i^4\} \end{vmatrix} \qquad (8)$$

$$[Y_k]^T := [E\{v_i\} \quad E\{v_i \psi_i\} \quad E\{v_i \psi_i^2\}]$$

A_k is described as a rotational, symbolically described as $\nabla_k \times (f_k \times \nabla_k)$, converging to zero, instead of being considered $\nabla_k \times (\nabla_k \times f_k)$, practically corresponding to interchange the last two rows of (8), so allowing the inverse an odd description. Here in a stochastic sense $\nabla_k := [E\{\psi_i^2\} \quad E\{\psi_i^3\} \quad E\{\psi_i^4\}]$, and $f_k := [E\{\psi_i\} \quad E\{\psi_i^2\} \quad E\{\psi_i^3\}]$, and the odd rotational $W_k := \nabla_k \times (\nabla_k \times f_k)$ is developed in (9). Without this property, the coefficients matrix has null values.

$$E\{\psi_i^4\}(E\{\psi_i\})^2 - 2E\{\psi_i\}E\{\psi_i^2\}E\{\psi_i^3\} + (E\{\psi_i^2\})^3 - kE\{\psi_i^4\}E\{\psi_i^2\} + k(E\{\psi_i^3\})^2 \tag{9}$$

Each parametre is described in (9), (10) and, (11) according to (1), (2) and, (3).

$$\hat{a}_k = \frac{\left((E\{\psi_i^3\})^2 + E\{\psi_i^2\}E\{\psi_i^4\}\right)E\{y_i\} + \left((E\{\psi_i\}E\{\psi_i^4\} - E\{\psi_i^2\}E\{\psi_i^3\})\right)E\{y_i\psi_i\} + \left((E\{\psi_i^2\})^2 - E\{\psi_i\}E\{\psi_i^3\}\right)E\{y_i\psi_i^2\}}{W_k} \tag{10}$$

$$\hat{b}_k = \frac{\left((E\{\psi_i\}E\{\psi_i^4\} - E\{\psi_i^2\}E\{\psi_i^3\})\right)E\{y_i\} + k\left(E\{\psi_i^2\}^2 - E\{\psi_i^4\}\right)E\{y_i\psi_i\} + \left((kE\{\psi_i^3\} - E\{\psi_i\}E\{\psi_i^3\})\right)E\{y_i\psi_i^2\}}{W_k} \tag{11}$$

$$\hat{c}_k = \frac{\left((E\{\psi_i^2\})^2 - (E\{\psi_i\}E\{\psi_i^3\})\right)E\{y_i\} + \left(E\{\psi_i^3\} - E\{\psi_i\}E\{\psi_i^2\}\right)E\{y_i\psi_i\} + k\left(E\{\psi_i\}^2 - E\{\psi_i^2\}\right)E\{y_i\psi_i^2\}}{W_k} \tag{12}$$

In symbolic form are (13), (14) and, (15).

$$\hat{a}_k = \frac{\left((T_k)^2 + Q_k U_k\right)P_k + \left((S_k U_k - Q_k T_k)\right)R_k + \left((Q_k)^2 - S_k T_k\right)W_k}{U_k(S_k)^2 - 2S_k Q_k T_k + (Q_k)^3 - kU_k Q_k + k(T_k)^2} \tag{13}$$

$$\hat{b}_k = \frac{\left((S_k U_k - Q_k T_k)\right)P_k + \left((Q_k)^2 - KU_k\right)R_k + \left((KT_k - S_k Q_k)\right)W_k k}{U_k(S_k)^2 - 2S_k Q_k T_k + (Q_k)^3 - kU_k Q_k + k(T_k)^2} \tag{14}$$

$$\hat{c}_k = \frac{\left[(Q_k)^2 - (S_k T_k)\right]P_k + \left((KT_k - S_k Q_k)\right)R_k + \left((S_k)^2 - (KQ_k)\right)W_k}{U_k(S_k)^2 - 2S_k Q_k T_k + (Q_k)^3 - kU_k Q_k + k(T_k)^2} \tag{15}$$

And considering stationary conditions the coefficients are described on (16) to (22).

$$P_k = \frac{1}{k}\left[y_k + (k-1)P_{k-1}\right] \tag{16}$$

$$Q_k = \frac{1}{k}\left[\psi_k^2 + (k-1)Q_{k-1}\right] \tag{17}$$

$$R_k = \frac{1}{k}\left[y_k \psi_k + (k-1)R_{k-1}\right] \tag{18}$$

$$S_k = \frac{1}{k}\left[\psi_k + (k-1)S_{k-1}\right] \tag{19}$$

$$T_k = \frac{1}{k}\left[\psi_k^3 + (k-1)T_{k-1}\right] \tag{20}$$

$$U_k = \frac{1}{k}\left[\psi_k^4 + (k-1)U_{k-1}\right] \tag{21}$$

$$W_k = \frac{1}{k}\left[y_k \psi_k^2 + (k-1)W_{k-1}\right] \tag{22}$$

These coefficients are applied in (4), (5) and, (6).∎

Simulation

The Ricatti observable SISO system in differences has the form (23).

$$y(k) = au(k) + by(k-1) + cy(k-1)^2 \tag{23}$$

In the first experiment, where $y(k)$ has a periodic function, the identifier is described in (24).

$$\hat{y}(k) = \hat{a}(k)u(k) + \hat{b}(k)y(k-1) + \hat{c}(k)y(k-1)^2 \quad (24)$$

And using (10), (11) and (12), the estimation applied in (24), and the identifier with the reference signal are scheme is shown in figure 1 and the simulation result is shown in figure 2.
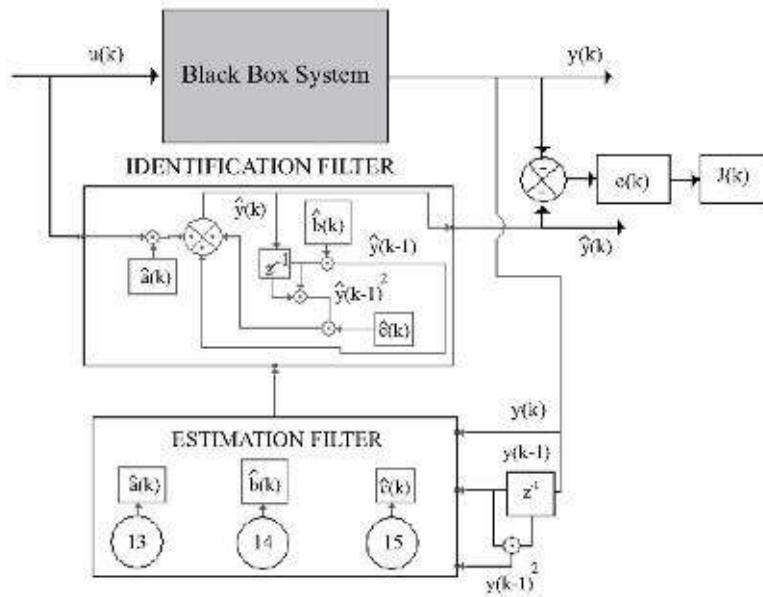

Figure 1 Estimation and identification filter with respect to input-output black box system

Where the error estimation is described by in agreement to figure 1 as $e(k)$ and its second probability moment as $J(k)$
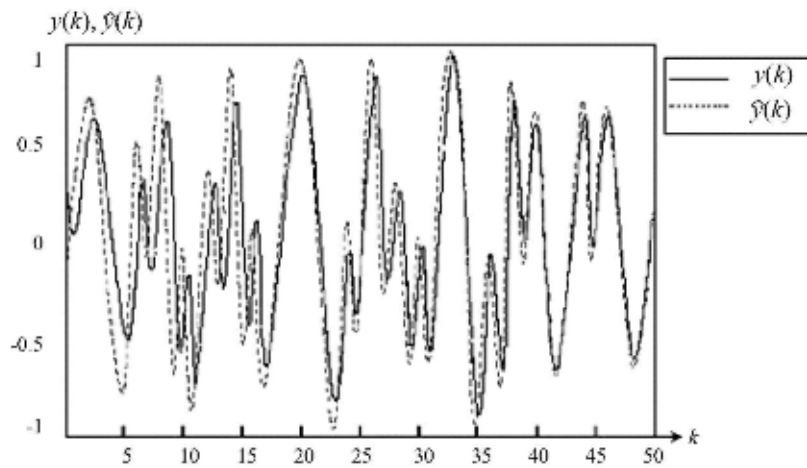

Figure 2 Identification results $\hat{y}(k)$ and reference signal with slow perturbation $y(k)$

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In the second experiment described by (24), the parametres are estimated based on (13) to (15), and described in figure 1, and depicted in figure 3, where the three parametres affected the reference signal.
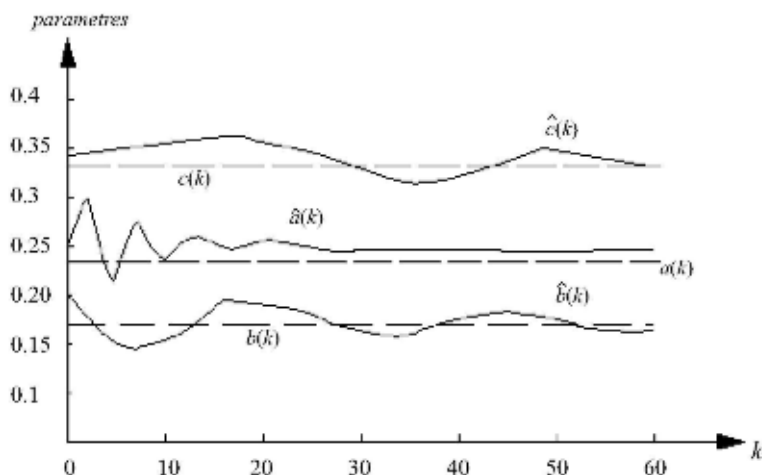

Figure 3 Parametres estimation and its references

The output observable signal with bounded random sequence is shown in figure 4, depicting the output identification evolution.
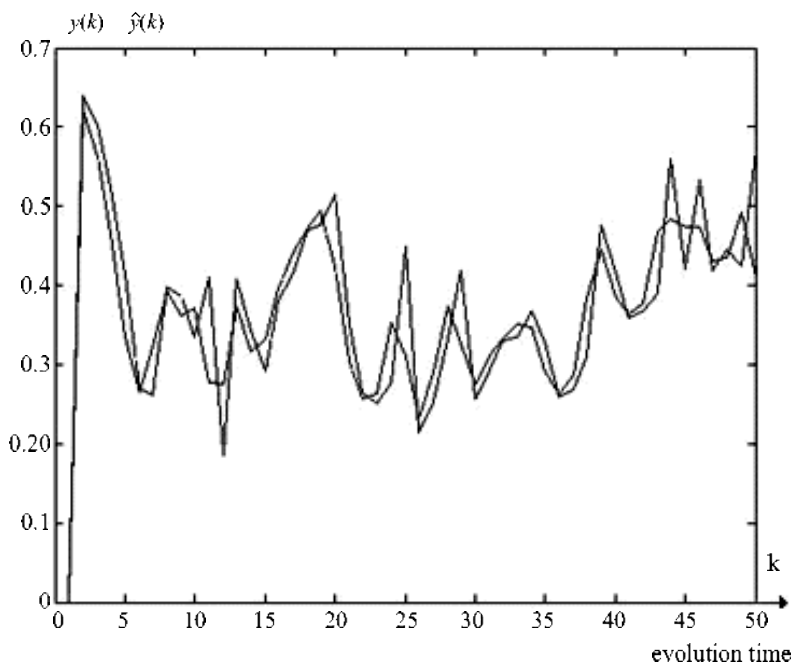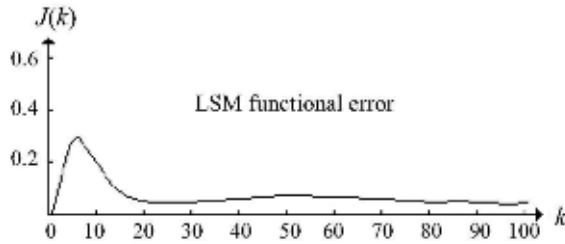

Figure 4 The reference *y(k) and* the identification $\hat{y}(k)$ signals *bounded*

Figure 5 shows LSM and Ricatti functional errors, respectively based on (25)

$$J(k) = \frac{1}{k}\left[e(k)^2 + (k-1)J(k-1)\right] \qquad (25)$$

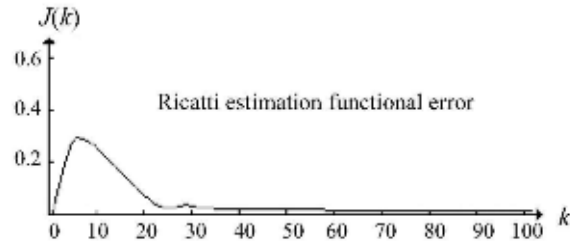The identification error is defined as $e(k) := y(k) - \hat{y}(k)$.

Figure 5 Functional errors

Conclusion

In this paper a stochastic digital filter as a parametre estimation shown by the Ricatti differences equation was proposed. A first difference order model with uncorrelated innovation conditions described in (1) as a SISO model in (23) considered two auxiliary equalities bounding the spatial operation, which allowed an approximation to a real parametres set. Three parametres were estimated, and the recursive forms from (13) to (15) considered stationary conditions listed from (16) to (22). The functional error (25) described illustratively the convergence rate shown in figure 5.

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