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Explicit pipe friction factor equations: evaluation, classification, and proposal

Ecuaciones explícitas del factor de fricción de tuberías: evaluación, clasificación y propuesta

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ABSTRACT: The Colebrook equation has been used to estimate the friction factor (f) in turbulent fluids. In this regard, many equations have been proposed to eliminate the iterative process of the Colebrook equation. The goal of this article was to perform an evaluation, classification, and proposal of the friction factor for better development of hydraulic projects. In this study, Gene Expression Programming (GEP), Newton-Raphson, and Python algorithms were applied. The accuracy and model selection were performed with the Maximum Relative Error $(\Delta f/f)$, Percentage Standard Deviation (PSD), Model Selection Criterion (MSC), and Akaike Information Criterion (AIC). Of the 30 equations evaluated, the Vatankhah equation was the most accurate and simplest to obtain the friction factor with a classification of very high, reaching a value of $\Delta f/f < 0.5\%$ and $1.5 < \mathrm{PSD} < 1.6$ A new equation was formulated to obtain the explicit f with fast convergence and accuracy. It was concluded that the combination of GEP, error theory, and selection criteria provides a more reliable and strengthened model.

RESUMEN: La ecuación de Colebrook se ha utilizado para estimar el factor de fricción (f) en fluidos turbulentos. En este sentido, se han propuesto varias ecuaciones para eliminar el proceso iterativo de la ecuación Colebrook. El objetivo de este artículo fue realizar una evaluación, clasificación y propuesta del factor de fricción para un mejor desarrollo de proyectos hidráulicos. En este estudio, se aplicaron los algoritmos de programación de expresión génica (GEP), Newton-Raphson y Python. La precisión y la selección del modelo se realizaron con el Máximo Error Relativo $(\Delta f/f)$, Porcentaje de Desviación Estándar (PSD), Criterio de Selección del Modelo (MSC) y Criterio de Información de Akaike (AIC). De las 30 ecuaciones evaluadas, la ecuación de Vatankhah fue la más precisa y sencilla para obtener el factor de fricción con una clasificación de muy alta, alcanzó un valor de $\Delta f/f < 0.5\%$ y $1.5 < \mathrm{PSD} < 1.6$ Se formuló una nueva ecuación para obtener el f explícita con rápida convergencia y precisión. Se concluyó que la combinación de GEP, teoría del error y criterios de selección proporciona un modelo más confiable y fortalecido.

1. Introduction

Pipes are used worldwide for the transportation of liquids with different properties. Non-Newtonian fluids are transported in pipelines in the mining and metallurgical industries, such as drilling mud, cementitious composites, and pastes [1]. In contrast, Newtonian fluids have a wider field of use, especially in turbulent flow over rough surfaces, with several engineering applications such as industrial plants, internal distribution networks in buildings, hydraulic turbines, irrigation systems, and drinking water pipelines [2], as well as in open-channel hydraulics [3]. Head losses are common in pipes or open channels, an essential parameter that affects the design and operation of the circulation flow in hydraulic works [4, 5].

In piping systems, head losses are analyzed by the

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universal Darcy-Weisbach equation. However, the implicit friction factor (f) intervenes in the equation. In this sense, Colebrook [6] proposes an equation that is currently the best approximation of the friction factor, especially for turbulent flow [7]. Nevertheless, its calculation is complex and cumbersome because the friction factor is present at both ends of the equation. In addition, its solution needs more time and processing in calculators. Therefore, its solution requires using iterative methods such as the Newton-Raphson approximation method. Although diagnostic and control algorithms are implemented in the mathematical modeling of hydraulic systems, precise parameter tuning is necessary.

Several authors [8–15] have developed explicit approximations of the friction factor as an alternative to the Colebrook equation, but the explicit models developed differ in their accuracy and computational efficiency [16–19]. The work presented by [20] highlighted that the equation by [21] was more accurate than the Colebrook equation for the experimental data in their research. On the other hand, [22] cite that the equations by [16] and [23] are the most efficient, with a maximum-recorded error of 0.18% and 0.54%, respectively. Likewise, [24] propose that the equations available in the literature lead to a deviation of between 2% and 3% for a turbulent flow with a Reynolds number of 2300. In turn, they suggest a new equation based on the relationship between friction forces and viscous forces to determine f with a maximum standard deviation of 0.25% with respect to the Colebrook equation.

There have been significant contributions in recent years to predicting the friction factor value with artificial intelligence approaches such as Gene Expression Programming (GEP), Evolutionary Polynomial Regression (EPR), Adaptive Neuro-Fuzzy Inference System (ANFIS), Artificial Neural Network (ANN), and physical and numerical models that manage to predict the fluid behavior in different media [25–28]. In particular, [26] estimated f using Bayesian learning neural networks and reached a relative error of 0.0035%. Furthermore, [29], using some artificial intelligence approaches, reached mean absolute errors of 0.001%. In this sense, [30] cite some gaps in the artificial intelligence technique, such as the data set, the layers of predesignated neurons, the percentage of training, and the test in the model tree. However, increasing the number of variables and implicit functions of the friction factor is necessary. Likewise, there is still a need to insert model selection criteria.

Many authors tend to use the Mean Squared Error (MSE), Mean Relative Error (MRE), Mean Absolute Error (MAE), Standard Deviation (SD), and Relative Error (RE) [7, 22] and [31]. This has several disadvantages when compared to other models since the value of R is more significant when the number of variables in the mathematical model increases [32]. Therefore, the value R can be increased, and the models can become more complex.

There are several techniques to adjust the training error for model sizes, such as Model Selection Criteria (MSC), Akaike Information Criterion (AIC) [33], Bayesian Information Criterion (BIC) [34], and Mallows' Cp Criterion [35]. The MSC and AIC have applied for the best prediction model, but there have been limits: $4000 < \mathrm{Re} < 108$ and $10^{-6} < \mathrm{e/D} < 5 \cdot 10^{-2}$ [12, 36], discrepancies in the results. The selection is important because the decision of the criterion could affect the interpretation of the variable as well as its prediction. Thus, the following hypothesis is proposed in the present study: the explicit friction factor equations can be classified, and the GEP can provide a new equation with a minimum error. In this sense, the goal of this work was to perform an evaluation, classification, and a new suggested explicit pipe friction factor equation with the least amount of error.

2. Materials and methods

The Colebrook equation is the most cited, accepted, and validated equation in fluid dynamics studies for obtaining friction losses in pipes.
It relates, in its implicit form, to the unknown friction factor (f), the relative roughness (e/D), the known pipe inner surface area, and the known Reynolds number (Re). Valid for $4000 < \mathrm{Re} < 10^8$ y $0 < \mathrm{e/D} < 5 \cdot 10^{-2}$, as shown in Equation 1. However, Equation 1 requires some mathematical iterations to get the optimal solution.

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon}{3.71D} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right) \qquad (1)$$

Where f is the implied friction factor (f), e is the absolute roughness of the pipe's inside wall, D is the pipe diameter, and Re is the Reynolds number.
Nonetheless, there are several explicit approaches reported in the scientific literature to calculate the friction factor, as shown in Equations 2 to 36.
[21].

$$f = \left[-2\log\left(\frac{\varepsilon/D}{3.7} + \frac{4.5}{\mathrm{Re}}\log\frac{\mathrm{Re}}{6.97}\right)\right]^{-2} \qquad (2)$$

$\mathrm{Re} \geq 10^4$ and $0 < \varepsilon/\mathrm{D} < 5 \cdot 10^{-2}$

[24]

$$f = \left[-2\log\left(\frac{\varepsilon}{3.7D} + \frac{10.04}{\mathrm{R}^*}\right)\right]^{-2} \qquad (3)$$

$$R^* = 2\,\mathrm{Re}\left[-\log\left(\frac{\varepsilon/D}{3.7} + \frac{5.45}{\mathrm{Re}^{0.9}}\right)\right]^{-1} \qquad (4)$$

R* the dimensionless number;
Limit: $\mathrm{Re} \geq 2300$ and $0 < \varepsilon/D < 0.05$

[37]

$$f=0.11\left[\frac{\varepsilon}{D}+\frac{68}{\mathrm{Re}}\right]^{0.25} \tag{5}$$

Limit: not specified

[38]

$$f=\frac{6.4}{\left[\ln(\mathrm{Re})-\ln\left(1+0.01\,\mathrm{Re}\,\frac{\varepsilon}{D}\left(1+10\sqrt{\frac{\varepsilon}{D}}\right)\right)\right]^{2.4}} \tag{6}$$

Limit: $\mathrm{Re}\leq 4000$

[39]

$$f=\left\{-2\log\left[\left(\frac{\varepsilon}{3.7D}\right)+\left(\frac{4.518\log\left(\frac{\mathrm{Re}}{7}\right)}{\mathrm{Re}\left(1+\frac{1}{29}\left(\mathrm{Re}^{0.52}\right)\left(\frac{\varepsilon}{D}\right)^{0.7}\right)}\right)\right]\right\}^{-2} \tag{7}$$

Limit: $5000<\mathrm{R_e}<10^{8}$ and $10^{-2}<\varepsilon/\mathrm{D}<10^{-6}$

[14]. Model I

$$f=\left[-2\log\left(10^{-0.4343\beta}+\frac{\varepsilon/D}{3.71}\right)\right]^{-2} \tag{8}$$

Limit: not specified

[14]. Model II

$$f=\left[-2\log\left(\frac{2.18\beta}{\mathrm{Re}}+\frac{\varepsilon/D}{3.71}\right)\right]^{-2} \tag{9}$$

Where β is:

$$\beta=\ln\left[\frac{\mathrm{Re}}{1.816\ln\left\{\frac{1.1\mathrm{Re}}{\ln(1+1.1\mathrm{Re})}\right\}}\right] \tag{10}$$

Limit: not specified

[17]

$$f=\left[0.8686\left(B+\left(\frac{1.038\ln(B+A)}{(0.332+B+A)}\right)-\ln(B+A)\right)\right]^{-2} \tag{11}$$

$$A=\left(\frac{\mathrm{Re}(\varepsilon/D)}{8.0878}\right) \tag{12}$$

$$B=\ln\left(\frac{\mathrm{Re}}{2.18}\right) \tag{13}$$

[40]

$$f=\left[-2\log\left(\frac{\varepsilon}{3.7065D}-\frac{5.0452}{\mathrm{Re}}\log\left[\frac{(\varepsilon/D)^{1.1098}}{2.8257}+5.8506\mathrm{Re}^{-0.8981}\right]\right)\right]^{-2} \tag{14}$$

Limit: $4000<\mathrm{Re}<10^{8}$ and $10^{-6}<\varepsilon/D<5\cdot 10^{-2}$

[41]

$$f=\left[-2\log\left(\frac{\varepsilon}{3.71D}+\frac{7}{\mathrm{Re}^{0.9}}\right)\right]^{-2} \tag{15}$$

Limit: $4000<\mathrm{Re}<10^{8}$ and $10^{-6}<\mathrm{E/D}<5\cdot 10^{-2}$

[42]

$$f=8\left[\left(\frac{8}{\mathrm{Re}}\right)^{12}+\left[\left[2.457\ln\left(\frac{1}{\frac{7^{0.9}}{\mathrm{Re}}+0.27\left(\frac{\varepsilon}{D}\right)}\right)\right]^{16}+\left(\frac{37530}{\mathrm{Re}}\right)^{16}\right]^{\frac{-3}{2}}\right]^{\frac{1}{12}} \tag{16}$$

Limit: $4000<\mathrm{Re}<10^{8}$ and $10^{-6}<\varepsilon/D<5\cdot 10^{-2}$

[43]

$$f=\left[-2\log\left(\frac{\varepsilon}{3.715D}+\frac{15}{\mathrm{Re}}\right)\right]^{-2} \tag{17}$$

Limit: not specified

[13]

$$f=\frac{0.2479-0.0000947(7-\log\mathrm{Re})^{4}}{\left[\log\left(\frac{\varepsilon}{3.615D}+\frac{7.366}{\mathrm{Re}^{0.9142}}\right)\right]^{2}} \tag{18}$$

Limit: not specified

[23]

$$f=1.613\left\{\ln\left[0.234\left(\frac{\varepsilon}{D}\right)^{1.1007}-\frac{60.525}{\mathrm{Re}^{1.1105}}+\frac{56.291}{\mathrm{Re}^{1.0712}}\right]\right\}^{-2} \tag{19}$$

Limit: $3000<\mathrm{Re}<10^{8}$ and $10^{-6}<\varepsilon/\mathrm{D}<5\cdot 10^{-2}$

[44]

$$f=\left\{-1.52\log\left[\left(\frac{\varepsilon}{7.21D}\right)^{1.042}+\left(\frac{2.731}{\mathrm{Re}}\right)^{0.9152}\right]\right\}^{-2.169} \tag{20}$$

Limit: $2100<\mathrm{Re}<10^{8}$ and $10^{-6}<\varepsilon/\mathrm{D}<5\cdot 10^{-2}$

[45]

$$f=\left\{-1.8\log\left[\left(\frac{\varepsilon}{3.7D}\right)^{1.11}+\frac{6.9}{\mathrm{Re}}\right]\right\}^{-2} \tag{21}$$

Limit: $400<\mathrm{Re}<10^{8}$ and $10^{-6}<\varepsilon/D<5\cdot 10^{-2}$

[46]

$$f=\left[-2\log\left(\frac{\varepsilon}{3.7D}+\frac{95}{\mathrm{Re}^{0.983}}-\frac{96.82}{\mathrm{Re}}\right)\right]^{-2} \tag{22}$$

Limit: $5235<\mathrm{Re}<10^{9}$

[47]

$$f=\left\{-1.8\log\left[\frac{7.35-1200(\varepsilon/D)^{1.25}}{\mathrm{Re}}+\left(\frac{\varepsilon}{3.15D}\right)^{1.115}\right]\right\}^{-2} \tag{23}$$

Limit: $4000 \leq \mathrm{Re} \leq 35.5 \cdot 10^6$

[12]

$$f = \left\{-2\log\left[\frac{\varepsilon}{3.7065D} - \frac{5.0272}{\mathrm{Re}}\log\left(\frac{\varepsilon}{3.827D} - \frac{4.657}{\mathrm{Re}}\log\left(\left(\frac{\varepsilon}{7.7918D}\right)^{0.9924} + \left(\frac{5.3326}{208.815+\mathrm{Re}}\right)^{0.9345}\right)\right)\right]\right\}^{-2} \quad (24)$$

Limit: $\mathrm{Re} > 4000$

[9]

$$f = 0.0055\left(1 + \left(2\cdot 10^4\left(\frac{\varepsilon}{D}\right) + \left(\frac{10^6}{\mathrm{Re}}\right)\right)^{\frac{1}{3}}\right) \quad (25)$$

Limit: $4000 < \mathrm{Re} < 10^8$ and $0 < \varepsilon/D < 10^{-2}$

[31]

$$f = \left\{-2\log\left[\left(\frac{\varepsilon}{3.71D}\right) - \frac{1.975}{\mathrm{Re}}\left(\ln\left(\left(\frac{\varepsilon}{3.93D}\right)^{1.092} + \left(\frac{7.627}{395.9+\mathrm{Re}}\right)\right)\right)\right]\right\}^{-2} \quad (26)$$

Limit: not specified

[48]

$$f = \left\{-2\log\left[\frac{\varepsilon}{7D} + \left(\frac{6.81}{\mathrm{Re}}\right)^{0.9}\right]\right\}^{-2} \quad (27)$$

Limit: $4000 < \mathrm{Re} < 10^8$ and $10^{-6} < \varepsilon/\mathrm{D} < 10^{-2}$

[49]

$$f = \left\{-1.8\log\left[0.27(\varepsilon/D) + \left(\frac{6.5}{\mathrm{Re}}\right)\right]\right\}^{-2} \quad (28)$$

Limit: $4000 < \mathrm{Re} < 10^7$ and $10^{-6} < \varepsilon/\mathrm{D} < 10^{-2}$

[50]

$$f = \left\{-2\log\left[\frac{\varepsilon}{3.7D} - \frac{5.02}{\mathrm{Re}}\log\left(\frac{\varepsilon}{3.7D} + \frac{14.5}{\mathrm{Re}}\right)\right]\right\}^{-2} \quad (29)$$

Limit: $4000 < \mathrm{Re} < 10^8$ and $10^{-6} < \varepsilon/D < 5\cdot 10^{-2}$

[51]

$$f = \left[-2\log\left(\left(\frac{\varepsilon}{3.715D}\right) + \left(\frac{6.943}{\mathrm{Re}}\right)^{0.9}\right)\right]^{-2} \quad (30)$$

Limit: $5000 < \mathrm{Re} < 10^8$ and $10^{-6} < \varepsilon/\mathrm{D} < 5\cdot 10^{-2}$

[10]

$$f = 0.25\left[\log\left(\frac{\varepsilon}{3.7D} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^{-2} \quad (31)$$

Limit: $5000 < \mathrm{Re} < 10^8$ and $10^{-6} < \varepsilon/D < 5\cdot 10^{-2}$

[16]

$$f = 0.8686\ln\left[\frac{0.3984\mathrm{Re}}{(0.8686S)^{\frac{S-0.645}{S+0.39}}}\right]^{-2} \quad (32)$$

Where S is:

$$S = 0.12363\mathrm{Re}(\varepsilon/D) + \ln(0.3984\mathrm{Re}) \quad (33)$$

Limit: not specified

[52]

$$f = 0.094(\varepsilon/D)^{0.225} + 0.53(\varepsilon/D) + 88(\varepsilon/D)^{0.44}\,\mathrm{Re}^{-1.62(\varepsilon/D)^{0.134}} \quad (34)$$

Limit: $\mathrm{Re} > 4000$ and $10^{-5} < \varepsilon/D < 5\cdot 10^{-2}$

[11] Model I.

$$f = \left\{-2\log\left[\left(\frac{\varepsilon}{3.7D}\right) - \frac{5.02}{\mathrm{Re}}\log\left(\frac{\varepsilon}{3.7D} - \frac{5.02}{\mathrm{Re}}\log\left(\frac{\varepsilon}{3.7D} + \frac{13}{\mathrm{Re}}\right)\right)\right]\right\}^{-2} \quad (35)$$

Limit: $4000 < \mathrm{Re} < 10^8$ and $10^{-5} < \varepsilon/\mathrm{D} < 5\cdot 10^{-2}$

[11] Model II.

$$f = \left\{-2\log\left[\left(\frac{\varepsilon}{3.7D}\right) - \frac{5.02}{\mathrm{Re}}\log\left(\frac{\varepsilon}{3.7D} + \frac{13}{\mathrm{Re}}\right)\right]\right\}^{-2} \quad (36)$$

Limit: $4000 < \mathrm{Re} < 10^8$ and $10^{-5} < \varepsilon/D < 5\cdot 10^{-2}$

The Colebrook equation and the 30 explicit equations found in the scientific literature were evaluated for different conditions of relative roughness (e/D) from 10^{-6} to $5\cdot 10^{-2}$ and the Reynolds number from 4000 to 108, which implied a base of 47601 data points. The analysis interval integrates the onset of turbulence and complete turbulence to test the best behavior of the correlations in the mathematical formulations.

In this study, the Newton-Raphson method was used in Colebrook Equation 1 by the Python algorithm. The method has been generalized due to its simplicity and speed of convergence to solve nonlinear problems, systems of equations, and nonlinear differential and integral equations [23].

Similarly, Gene Expression Programming (GEP), implemented in GeneXpro software, was applied, after obtaining the evaluation, classification, and generation of the most suitable equations. Initially, the database composed of 47,601 variables was used to select the best adjustment according to their fitness and introduce genetic variation using genetic operators.

Additionally, the procedure for estimating the pipeline friction coefficient using GEP involved fitness function selection, choice of T-termini and F-functions to create chromosomes, choice of chromosome architecture, choice of linkage function, and choice of genetic operators.

The 30 Chromosomes were executed, with a head size of 8 and the number of genes 1, 2, 3, and 6; linking functions (+, -, *, /); and mathematical functions divided into GEP1, GEP2, GEP3, and GEP4 in $+,-,/,\cdot,\sqrt{x},e^{x},\log_{10},10^{x}x^{1/3}$, $x^{1/4},x^{1/5},x^{2},x^{3},x^{4},x^{5},1/x$.

In the investigation, percent standard deviation (PSD) and Equation 37 Maximum Relative Error $(\Delta f/f)$ were used as criteria for the accuracy of the explicit models.

$$\frac{\Delta f}{f}=\left(\frac{f_{CW}-f_{\text{proposed}}}{f_{CW}}\right)100\% \quad (37)$$

Additionally, efficient methods of model comparison and selection based on model complexity were applied. Model Selection Criteria (MSC) [29] and Akaike's Information Criteria (AIC) were used [26]. These criteria expressed by Equations 38 and 39 are based on the greatest likelihood and smallest parameters, and the variables follow a normal distribution.

$$MSC=\ln\left[\frac{\sum_{i=1}^{n}\left(f_{CW}-\bar{f}_{\text{Proposed}}\right)^{2}}{\sum_{i=1}^{n}\left(f_{CW}-f_{\text{Proposed}}\right)^{2}}\right]-\frac{2p}{n} \quad (38)$$

$$AIC=n\ln\left[\frac{1}{n}\sum_{i=1}^{n}\left(f_{CW}-f_{\text{Proposed}}\right)^{2}\right]+2p \quad (39)$$

Where f_{CW} is the true value of the Colebrook-White (CW) friction factor, f_{proposed} is the value of the proposed friction factor, *p* is the number of equation parameters including constants, *i* = 1,... n is the number of friction factor values, and n is the sample size.

3. Results and discussion

Figure 1 shows the accuracies of the explicit models according to the Maximum Relative Error $(\Delta f/f)$ and percent standard deviation (PSD). Figure 1 a) shows that the $(\Delta f/f)$ values ranged from 0.082% to 38.435%, and 43% of the equations had values lower than 2.0% of the Maximum Relative Error. Group I is the most efficient approximation where the Maximum Relative Error is less than 1%; therefore, those are recommended for precision engineering work. In Group I, the results are outstanding, presenting values $\Delta f/f<0.5\%$ by the equations of [12], model I [11, 16, 31] and [17, 23, 24, 40]. In particular, the equations by [13, 39] and [50] have $0.5<\Delta f/f<1\%$.

Other authors have formulated new, noteworthy, accurate equations; these are classified in group II because they have a Maximum Relative Error of less than 2%, which are those proposed for model II by [11] and [45].

Group III was classified as having a lower approximation to Colebrook's with a Maximum Relative Error between 2.587 $2.587 \leq \Delta f/f \leq 8.303$, as equations cited by [21, 46], model II by [10, 14, 38, 44, 51], and model I by [14]. However, the equation by [21], according to [20] in their research, was the most accurate. Possible causes were that [20] only used 2397 experimental points, $3000\leq \mathrm{R\,e}\leq 735-10^{3}$, and 0 $0<\varepsilon/D<1.4-10^{-3}$. Nonetheless, group IV had to be rejected because they exceeded $\Delta f/f>$ 10%, as are [9, 42, 47–49, 52], and [37]. In particular, the equation proposed by [9], at the time provided significant results for solving problems, but it is shown that new and more accurate formulations have been developed.

Results that agree with those obtained by [22], who evaluated 33 equations in a range of the Moody diagram with $2300\leq \mathrm{Re}\leq 10^{8}, 0<\varepsilon/D<5-10^{-2}$ and in relation to the equation proposed by [9] the error test was high, exceeding 10%. Similarly, it agrees with the results by [29] on the mathematical models analyzed using Machine Learning tools in which [9] and [42] had the most unfavorable equations.

Regarding Figure 1 b) and the Percent Standard Deviation (PSD), it is observed that, in general, the 30 equations analyzed presented a deviation between 1.2%<PSD<2%. However, 81% of the equations had a stable standard deviation between 1.5% and 1.6%. Nevertheless, there are three equations of approximations with the lowest standard deviation, such as [9, 48] and [37], but they presented a high relative error for which they were rejected.

The 30 equations analyzed in this article have two perspectives: firstly, the equations with a high number of parameters tend to be more accurate, and secondly, the equations with the least number of parameters are less accurate. On the other hand, the engineer needs the easiest and most accurate equation for friction factor calculation, according to [24]. In summary, as a result of the increasing digitization of work, educational and economic environments, the equations must be formulated with the highest precision and best computational performance.

For this reason, the MSC and AIC Model Selection Criteria have been implemented using a Ranking because it considers a decisive variable as the number of parameters, including the constants in the equations *(p)*.

Based on the accuracies of the models, a preliminary model ranking (Rk) was proposed for each evaluation criterion $p,\Delta f/f,\mathrm{PSD},\mathrm{MSC}$, and AIC, and finally, a Global Ranking. Table 1 shows the results of the models. It is observed that the error theory and theoretical functions show results that differ in their rank order

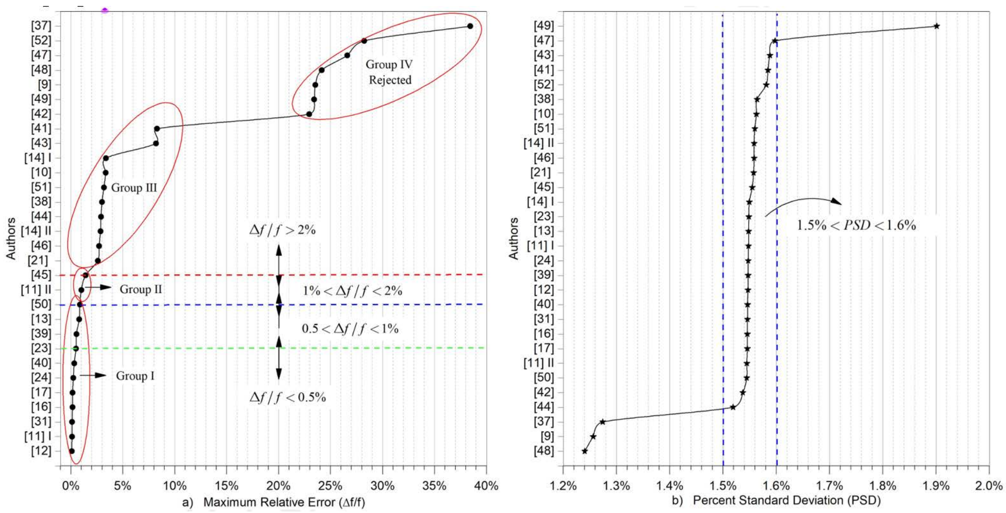

Figure 1 Comparative assessment flowchart

for each equation, with a discrepancy in optimal model selection. Equation 5, proposed by [37] is the simplest and has the least number of steps to obtain the friction factor. Nevertheless, in the previous analysis, it was rejected because of its high relative error, which is positioned at number 30. Meanwhile, Equation 11 by [17] is classified as the most complex for its solution due to the number of steps and parameters it includes. However, it was classified in group I with a relative error of less than 0.5% and an acceptable deviation of less than 1.6%, with a ranking of 8.

In this sense, MSC and AIC contributed to the selection of the best model. However, in both cases, they present discrepancies with respect to the function of greater likelihood and entropy. The MSC value indicates that by [49] equation occupies rank 1, while the MSC value of the [11] equation model I occupies rank 30. In relation, the AIC reached inversely proportional values, the [49] equation reached rank 30 and [11] equation model I has rank 1. On the other hand, in contrast to the previous equations, the number of parameters by [11] equation model I is 47% higher than by [49] equation. Consequently, it can be pointed out that the AIC criterion does not follow the parsimony principle because the smaller the number of parameters, the smaller the AIC tends to be.

contradict the theories for which the AIC criterion was defined. In finite samples, the AIC value is only approximate [33]. Therefore, difficulties could arise regarding the validity and applicability of the method for this purpose.

Additionally, the MSC criterion also showed inconsistencies between the models due to the number of parameters; however, this coincides with the results of the AIC criterion. This trend in the results corresponds with those results obtained by [36].

The global ranking obtained in Table 1 integrates the positions of the most accurate and inaccurate approximation models with their degrees of complexity. The explicit Equation 32 proposed by [16] leads the Global Ranking in the first position as the most accurate, followed in second place by Equations 29, 26, and 22 by [31, 50], and [46]. The least accurate and most complex to solve are Equations 34, 23, 28 by [47, 52], and [49], which in turn belong to the rejected group IV.

Consequently, in Table 1 an easier classification has been established, according to the level of precision and simplicity for the first five global rankings. It was established from a very high level, which indicates excellent precision and simplicity, to a very low level, which is interpreted as an inaccurate and complex equation to solve due to the number of operations and parameters present.

As a new proposal for explicit friction factor approximation equations, 64 models were analyzed in Gene Expression Programming (GEP). The theoretical and experimental databases were developed as a training process to train the GEP algorithm. Twenty percent of the data was reserved for validation and the rest for calibration. Only the most efficient results of GEP1, GEP2, GEP3, and GEP4 according to the performance criteria are reflected in Table 3.

Table 3 shows that the most significant models had Linking Functions + and *, a Number of Chromosomes of 30, a Head Size of 8, and a Number of Genes of 2 and 6. The best-performing model was GEP1, with the lowest number of functions (4), and 7 parameters including constants.

Table 1 Preference models

Authors	No. equations	p	Main statistics		Model selection criteria			Global Ranking
		Parameter	$\Delta f/f$	PSD	MSC	AIC	Total	Global
		No	Rk	Rk	Rk	Rk	$\sum$ Rk	GR
[21]	2	11	14	20	17	14	76	14
[24]	3	17	6	14	28	3	68	7
[37]	5	6	30	3	3	28	70	9
[38]	6	14	18	25	10	21	88	20
[39]	7	19	9	13	23	8	72	11
[14]I	8	18	21	18	11	20	88	20
[14]II	9	19	16	22	15	15	87	19
[17]	11	39	5	8	25	6	83	18
[40]	14	16	7	11	24	7	65	5
[41]	15	9	23	27	8	23	90	22
[42]	16	19	24	5	5	27	80	17
[43]	17	8	22	28	9	22	89	21
[13]	18	14	10	16	21	10	71	10
[23]	19	13	8	17	22	9	69	8
[44]	20	10	17	4	12	19	62	3
[45]	21	10	13	19	18	13	73	13
[46]	22	10	15	2	16	16	59	2
[47]	23	12	28	29	6	25	100	24
[12]	24	21	1	12	29	2	65	5
[9]	25	8	26	2	4	26	66	6
[31]	26	15	3	10	27	4	59	2
[48]	27	7	27	1	2	29	66	6
[49]	28	8	25	30	1	30	94	23
[50]	29	11	11	6	20	11	59	2
[51]	30	9	19	23	14	17	82	15
[10]	31	8	20	24	13	18	83	16
[16]	32	13	4	9	26	5	57	1
[52]	34	16	29	26	7	24	102	25
[11]I	35	17	2	15	30	1	65	5
[11]II	36	14	12	7	19	12	64	4

Table 2 Model classification

GR	Authors	Precision	Simplicity
1	[16]	Very high	Very high
2	[50]	Very high	High
2	[31]	Very high	High
3	[46]	Medium	Medium
3	[44]	Medium	Medium
4	[11]II	High	Low
5	[11]I	Very high	Very Low
5	[12]	Very high	Very Low
5	[40]	Very high	Very Low

The Root Mean Square Error (RMSE) was 0.078%, the Mean Absolute Error (MAE) was 0.055%, the Pearson correlation coefficient (R) was 0.99873, the $\Delta f/f$ was 6.22%, and the PSD was 1.86%.

In contrast to the groups made in Figure 1 due to the maximum relative error, GEP1 was classified in group III because it was within the interval $2.5 \leq \Delta \mathrm{f/f} \leq 8.3$, this being an alternative to obtain the friction factor quickly and easily.
Although GEP4 has the highest R and a lower $\Delta f/f$, PSD, it is shown to be more significant for having a greater number of functions, according to [24]. In addition, the GEP4 model has a greater number of operations for its solution, making it less simple. Regarding the increase of functions, the Number of Chromosomes, Head Size, and Number of Genes showed a partial relationship to the results obtained by [51] that the GEP models increase with increasing functions.

Equation 40 is proposed as a new nonlinear model to determine the explicit friction factor coefficient with the lowest error without the existence of logarithmic functions, speed of calculation, or more accurate approximation in the turbulent flow regime. The Limit: $4000 < \mathrm{Re} < 10^8$ and $10^{-6} < \varepsilon/\mathrm{D} < 10^{-2}$.

$$GEP1 = 0.219[(0.028\varepsilon/D) + (0.896/R)]^{0.25} \tag{40}$$

4. Conclusions

Thirty explicit friction factor equations were analyzed on a base of 47601 theoretical and experimental data points and according, to the maximum relative error $(\Delta f/f)$, were classified into 4 groups: group I of $0.5\% < \Delta f/f$, group II of $0.5\% < \Delta f/f < 1\%$, group III of $1\% < \Delta f/f < 2\%$ and group IV $\Delta f/f > 2\%$. Group I includes the most accurate explicit friction factor equations, developed by [12], model I [11, 16, 17, 24, 31, 40] and [23]. In general, the Percentage Standard Deviation (PSD) was acceptable and comprised between $1.2\% < \mathrm{PSD} \leq 1.9\%$.

The MSC and AIC selection criteria contributed to the selection of the most accurate equations to estimate the friction factor, but they presented a discrepancy in likelihood and entropy. However, the number of parameters and operations of the equations *(p)* was a decisive variable in obtaining the global ranking of the 30 friction factor equations explicit in Table 2. In summary, the first five global rankings were classified by the most accurate and simple equations. Therefore, it was concluded that the estimates of the equation by [16] ranked very high in accuracy and simplicity for obtaining explicit friction factors. The [50] and [31] equations also presented very high performance. In contrast, the use

Table 3 Efficient model of the GEP

Model	Functions	Performance criteria				
		Training and Validations				
		RMSE (%)	MAE (%))	R	$\Delta f/f$ (%)	PSD
GEP1	$+,-,/,*,\sqrt{x},$	0.078	0.055	0.99873	6.22	1.86
GEP2	$+,-,/,*\sqrt{x},e^x,\log_{10},\ln$	0.114	0.093	0.99729	6.49	1.92
GEP3	$+,-,/,*,\sqrt{x},e^x,\log_{10},\ln,10^x,x^{1/3},x^{1/4},x^{1/5}$	0.080	0.064	0.99868	6.31	1.89
GEP4	$+,-,/,*,\sqrt{x},e^x,\log_{10},10^x x^{1/3},x^{1/4},x^{1/5},x^2,x^3,x^4,x^5,1/x$	0.089	0.067	0.99893	6.20	1.84

of the equations developed by [52], [47], and [49] is not recommended, and in the case of their use, they should be under specific conditions because they can produce inaccurate results. This new approach made it possible to observe that, under certain conditions, the Colebrook equation is not the most accurate at present.

With the GEP, it was possible to provide a new model to determine the explicit friction factor $f(\mathrm{R}, \varepsilon/\mathrm{D})$ with the lowest degree of complexity in the turbulent flow regime. It has an RMSE of 0.078%, an MAE of 0.055%, and an R of 0.99873. Compared with the Colebrook equation, it has more simplicity, fast convergence, less computational time, and a good relationship between accuracy and computational efficiency. From the analyzed equations of the explicit friction factor for turbulent flow, it was found that there are new equations with optimal efficiency indicators for the original equations that are cited, such as those by [9, 10] and [51]. In this regard, it is recommended to consider the mathematical models' new functions as more accurate explicit approximations.

The main finding of the research developed is the integration of statistical tools, Python algorithms, Genetic Expression Programming, and the new model proposed for obtaining the level of complexity and effectiveness of the explicit friction factor equations of the Colebrook equation. Likewise, novel information would ease the elaboration and decision-making of hydraulic engineering projects. In response to the previous conclusion, it is recommended to extend the analysis methods with artificial intelligence and new criteria for the selection of mathematical models.

5. Declaration of competing interest

We declare that we have no significant competing interests, including financial or non-financial, professional, or personal interests, interfering with the full and objective presentation of the work described in this manuscript.

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8. Author contributions

Maiquel López Silva: Conceived and designed the analysis, the scientific literature review, the statistical analysis, and interpretation of data, the formulation of gene expression programming algorithms, prepared the text. Dayma Carmenates Hernández: Conceived and designed the analysis, assisted with the scientific literature review, statistical analysis, and interpretation of data, and prepared the text and edited the manuscript. Nancy Delgado Hernández: Scientific literature review, digital processing of data, formulation of gene expression programming algorithms, and Newton-Raphson programming in Python. Nataly Chunga Bereche: Scientific literature review, digital processing of data, formulation of gene expression programming algorithms, and Newton-Raphson programming in Python.

9. Data availability statement

The origin of the data is from the turbulent regime, with different conditions of relative roughness (/D) from 110^{-6} to 5×10^{-2} and the Reynolds number from 4000 to 108, which implied a base of 47601 data points. The authors confirm that the data supporting the findings of this study are available within the article and its supplementary materials.

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