Revista Facultad de Ingeniería, Universidad de Antioquia, No.113, pp. 106-113, Oct-Dec 2024

Regionalization of maximum daily Rainfall in the Boyacá department

Regionalización de la precipitación máxima diaria en el Departamento de Boyacá, Colombia

Laura Natalia Garavito-Rincón [1*], Carlos Andrés Caro-Camargo [1], Johana Catherine Casallas-Chaves [1]

[1]Facultad de Ingeniería Civil, Universidad Santo Tomás Tunja. Avenida Universitaria # 45-202. Tunja, Colombia.

CITE THIS ARTICLE AS:

L. N. Garavito-Rincón, C. A. Caro-Camargo, and J. C. Casallas-Chaves. "Regionalization of Maximum Daily Rainfall in the Boyacá department", *Revista Facultad de Ingeniería Universidad de Antioquia*, no. 113, pp. 106-113, Oct-Dec 2024. [Online]. Available: `https://www.doi.org/10.17533/udea.redin.20240308`

ARTICLE INFO:

Received: September 01, 2022
Accepted: March 03, 2024
Available online: March 03, 2024

KEYWORDS:

Rainfall; homogeneity; cluster; Andrews curve; region

Precipitación; homogeneidad; cluster; curva de Andrews; región

ABSTRACT: Climate change has generated alterations in hydrometeorological variables behavior, accentuating extreme events, especially those related to precipitation, which is a key phenomenon in the design and planning of territorial hydraulic structures. This research shows the rainfall regionalization in the department of Boyacá, Colombia, through the application of Cluster and Andrews Curve and validation of L-moments, as input for a hydroclimatological characterization that can feed hydrological methods or models in the region, resulting in 8 homogeneous areas of maximum annual precipitation..

RESUMEN: El cambio climático ha generado alteraciones en el comportamiento de las variables hidrometeorológicas acentuando los eventos extremos. Esta problemática ha incentivado un incremento en la ocurrencia de eventos macroclimáticos, especialmente los referidos a precipitaciones; fenómenos claves en el ordenamiento territorial y diseño de estructuras hidráulicas. Esta investigación muestra la regionalización de la precipitación en el departamento de Boyacá, Colombia, a través de la aplicación Cluster y Curva de Andrews y validación de L-momentos, como insumo para una caracterización hidroclimatológica que pueda alimentar métodos o modelos hidrológicos en la región, y teniendo como resultado 8 áreas homogéneas de precipitación máxima anual.

1. Introduction

Among the problems associated with the issue of water and climate change in Latin America, the decrease in rainfall regimes in arid areas, melting glaciers, and difficulties with water supply for domestic, agricultural, and energy production can be found [1]. This region has shown discontinuity and uncertainty in the reliability of historical climate series, which makes the spatio-temporal characterization of the variables complex to determine [2]. The reliability of hydroclimatological data is fundamental in adequately characterizing the hydrological reality and quantifying variables in contribution areas [3].

Climate action impacts human development, directly influencing health, economy, territorial planning, risk management, and migratory processes [4]. Water, as a limited resource, has decreased in quality over time; therefore, there is a need to implement strategies and tools to control water resources, such as hydrological models, ecological and climatic classification, flow evaluation, design, regulation scenarios, and hydraulic works [5, 6].

Likewise, control and planning tools for urban and rural basins are gaining importance in water resource planning [7].

* Corresponding author: Laura Natalia Garavito-Rincón
E-mail: secdiv_arq_ing@ustatunja.edu.co
ISSN 0120-6230
e-ISSN 2422-2844

DOI: 10.17533/udea.redin.20240308

One analysis source of these strategies is the knowledge of the homogeneous hydrological regions of a certain study area [5].

Hydrological or climatic regions relate to the union of zones that occupy a geographic area that are connected by their high degree of similarity in their hydrological response or climatic behavior. The regions are identified to group, synthesize, and determine information about areas that are not gauged, to be able to carry out analysis and diagnoses and to develop projects regarding the use of water resources, as well as to formulate programs based on water conservation, land use, planning, flood prevention evaluation, among others [8]. There are various regionalization methods for hydrometeorological variables: the following stand out: Principal Component or Cluster Analysis [9], potential multiple regression equations [2], partial least squares regression analysis [10], Andrews Curve, Seasonality Index, Classification Trees, Pattern Approach residual [11], Canonical Correlation [12].

This article consolidates the homogeneous hydrological regions in the Department of Boyacá using two methodologies: Cluster analysis, which consists of grouping a homogeneous variable [13], and Andrews Curves [14]. All of this aims to generate a regionalization proposal that allows the transferring of Precipitation information to non-instrumented areas. Regionalized climate series is a necessity in reliable hydrological modeling at a high-resolution level and for case studies in rural and urban basins [15].

2. Methodology

The study area is the Department of Boyacá, Colombia, located in central Colombia. Its harsh topography generates the existence of variability in its hydroclimatological conditions, with the presence of all thermal levels and temperatures. In some parts of its extension, some areas are not gauged, generating insufficient knowledge of hydrological behavior [16], as well as the existence of hydroclimatological series that do not comply with the parameters established by the World Meteorological Organization (WMO).

The stations within and around the study area, with information on annual Rainfall maximums, were provided by the IDEAM (Institute of Hydrology, Meteorology and Environmental Studies, Colombia). The definition of the study period for precipitation was based on the periods suggested by the WMO, using at least 30 years of precipitation data [12], considering that the time series should not be too short, avoiding increasing the estimation errors of statistical parameters such as the quartiles, producing the wrong selection of stations [17]. Additionally, the aforementioned selection of study period entirely agrees with the provision of Colombian legislation associated with risk management policies in the country [18]. Prior to the hydrometeorological regionalization study, the pre-processing of the unified historical series of maximum monthly precipitation is accomplished through the identification of tests of independence, seasonality, and stability in the mean of homogeneity. The above uses the Grubbs-Beck, Waldwolfowitz, and Mann–Whitney tests, respectively.

Regionalization by Cluster or cluster analysis

Statistical cluster analysis has been effectively used in climatic and atmospheric studies, mainly in determining homogeneous climatic regions based on historical meteorological series [5]. This exploratory cluster analysis splits the daily Maximum Rainfall data series into sets with similar characteristics. Most methods rely on the distance between the means of two groups being greater than the mean distance within a group [12]. The main clustering algorithms are hierarchical, non-hierarchical, and combined [6, 19].

The non-hierarchical technique, often called k-means clustering [6], is more flexible than hierarchical clustering [19]. The method consists of preselecting the number of clusters k, randomly choosing the centroids or time series of each group, assigning grid cells to the most similar centroid, recalculating centroids by averaging all-time series assigned to that centroid, and iterating until convergence.

Hierarchical Clusters consist of constructing a tree-shaped structure called a dendrogram, where the number of groupings gradually increases or decreases depending on the distance defined by the expert. The ideal cutting distance corresponds to the height of a visible number of groups [19, 20]. This method is helpful in exploratory analyses and has been preferred in studies defining climate regions [5].

For the region under study, the Hierarchical Clustering, Centroid, and Ward methods [20] are applied.

The first hierarchical method worked was Ward's or minimum variance, which groups with a minimum information loss criterion, based on sums of squares [21]. The method starts at zero and then grows as groups are merged [5].

The second method was Within, which determines the distance between two clusters as the average of the distances between all cluster pairs [22], thus measuring their homogeneity [23].

The cut level in the dendrogram is defined arbitrarily through the choice of the distance criterion method, considering that this is a measure of similarity between elements. It indicates that the greater the magnitude, the lower the probability they belong to the same group [24]. The cluster extension validity index used is the Euclidean distance (d), defined in the Equation (1):

$$d = 1 - \frac{\text{Pearson correlation}}{\text{Time series (Two sources)}} \quad (1)$$

A higher correlation is equivalent to a smaller Euclidean distance and dictates probable clustering [6].

Regionalization by Andrews curve

This method corresponds to a graphical approach developed by Andrews (1972), implemented to establish similarity patterns in multiple dimensions for certain variables. A point in multidimensional space is represented in a curve as described by the function seen in Equation (2) [14]:

$$f(t) = \frac{x_1}{\sqrt{2}} + x_2\sin(t) + x_3\cos(t) + x_4\sin(2t) + x_5\cos(2t) + x_6\sin(3t) + x_7\cos(3t)\ldots \quad (2)$$

Where:

f(t)= Harmonic function of the individual (multidimensional point)
x1, x2, x3... xn = Individual variables (Maximum annual Rainfall)
t= Variable of the harmonic function $(-\pi \text{a} \pi))$

The difference between the two curves, each corresponding to its behavior in the maximum annual Rainfall series, will mean a greater or lesser degree of homogenization. Taking the above into account, groups of similar stations are visually identified by representing them as a band of spatially close curves [13, 25] that allow the generation of a region.

Validation of homogeneity of regions through L - Moments

The homogeneity of the groups and identification of regional distribution for each group is validated using the L-Moments method [26]. They are defined as linear functions of probability-weighted moments, robust to outliers and unbiased for small samples [27]. Where the data (x1: n) are initially sorted in ascending order of 1 and the first four probability weighted moments (PWMs) formulated by Hosking and Wallis (1997) are determined, which are defined in Equations (3) to (5):

$$\left.\begin{array}{c} \beta_0 = \frac{1}{n}\sum_{j=1}^{n} x_j \\ \beta_1 = \frac{1}{n}\sum_{j=2}^{n} x_j\left[\frac{j-1}{n-1}\right] \\ \beta_2 = \frac{1}{n}\sum_{j=3}^{n} x_j\left[\frac{(j-1)*(j-2)}{(n-1)*(n-2)}\right] \\ \beta_3 = \frac{1}{n}\sum_{j=4}^{n} x_j\left[\frac{(j-1)*(j-2)*(j-3)}{(n-1)*(n-2)*(n-3)}\right] \end{array}\right\} \quad (3)$$

The L moments are defined as set out in Equation (4) [28]:

$$\left.\begin{array}{c} L_1 = \beta_0 \\ L_3 = 6\beta_2 - 6\beta_1 + \beta_0 \\ L_2 = 2\beta_1 - \beta_0 \\ L_4 = 20\beta_3 - 30\beta_2 + 12\beta_1 - \beta_0 \end{array}\right\} \quad (4)$$

Where:

L1= average
L2 = degree of dispersion or L-Cv
L3 = asymmetry or L-bias
L4 = kurtosis or L-Kurtosis
L-coefficients of variation, skewness and kurtosis, are defined by Equation (5) [28]:

$$\left.\begin{array}{c} \text{Mean } (t_1) = L_1 \\ L - Cv\,(t_2) = \frac{L_2}{L_1} \\ L - \text{asymmetry}\,(t_3) = \frac{L_3}{L_2} \\ L - \text{ kurtosis }(t_4) = \frac{L_4}{L_2} \end{array}\right\} \quad (5)$$

Table 1 comprehensively lists the guidelines for characterizing the relative magnitude of L- Cv and L – skewness for a data series, magnitudes that should be similar in a homogeneous region.

Discordance tests of defined regions are used to analyze regions whose L moment relationships are discordant with the overall data [28]. Regions with a discordance measure greater than 3 are considered discordant in relation to the collective behavior of the proposed grouping, are defined by Equation (6) and (7) [27].

$$D_i = 1/3\,(u_i - U)^T * A^{-1} * (u_i - U) \quad (6)$$

Where:

$$\left.\begin{array}{c} u_i = \left[t^i t_2{}^i t_3{}^i\right]^T \\ \text{U} = \frac{1}{n}\sum_{i=1}^{n} u_i \\ \text{A} = \frac{1}{n-1}\sum_{i=1}^{n}(u_i - U)(u_i - U)^T \end{array}\right\} \quad (7)$$

N = number of stations in the region.
ui= Transposed vector of the product of t1, t2 and t3
U = station group average
A = sample covariance matrix

The critical range of the discordance indicator Di is a function of the number of stations that constitute the

Table 1 General descriptions of the relative magnitude of the L Moments

L- coefficients of variation		**L- asymmetry**	
Worth	Description	Worth	Description
0.0 < | t2 | < .025	Minimum variability	t3 = 0.0	Symmetrical distribution
0.25 < | t2 | < 0.075	Minor variability	0.0 < t3 ≤ 0.05	Minor Bias
0.075 < | t2 | < 0.15	Moderate Variability	0.05 < t3 ≤ 0.15	Moderate bias
0.15 < | t2 | < 0.4	Great Variability	0.15 < t3 ≤ 0.3	Big bias
0.4 < | t2 |	Very great variability	0.3 < t3	Very biased

Source: L-RAP User's Manual, 2011

region [28]. A homogeneous region is evident when its underlying distribution is the same for the stations that compose it. Therefore, it is necessary to analyze its degree of homogeneity or heterogeneity. Hosking & Wallis developed the heterogeneity measure for hydroclimatic regions H1, as an indicator of the degree of heterogeneity in the L-Moments-ratios for a group of stations. This statistical test measures the relative variability of the coefficient of Variation (L-CV), H2 measures the variability in L-Bias, and H3 measures the variability in L-Kurtosis [29], defining those shown in Table 2 as critical values.

Table 2 Critical ranges of heterogeneity

Heterogeneity	**Criterion**
Homogeneous	H<2
Possibly heterogeneous	2<H<3
Heterogeneous	H>3

Source: Wallis *et al.*, 2007

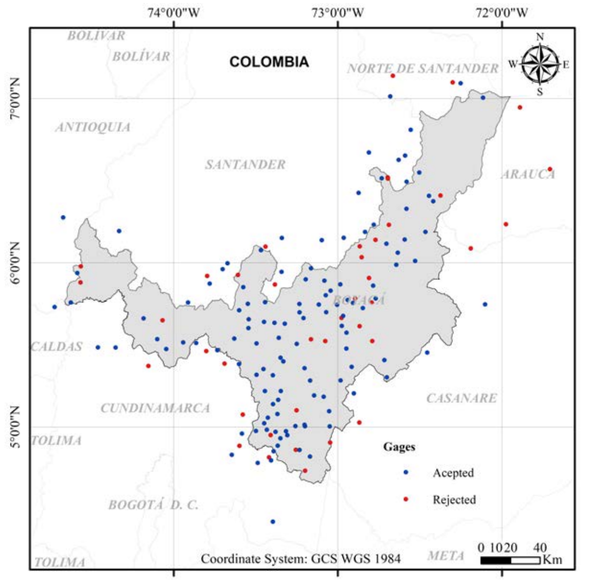

Figure 1 Geographic location of the stations

The weighted standard deviation of the L-CV is determined through the application of Equation (8):

$$\left.\begin{aligned} V &= \left\{ \frac{\sum_{i=1}^{N} \left(t^i - t^R\right)^2}{\sum_{i=1}^{N} N_i} \right\}^{\frac{1}{2}} \\ &\vdots \\ H_1 &= \frac{V - \mu_v}{\sigma_v} \end{aligned}\right\} \quad (8)$$

Where:

μv = standard average V

σv= standard deviation V

H1 = homogeneity indicator

2.1 Results

In the definition of hydroclimatological regions of the department of Boyacá, the range between 1984 and 2018 was taken as the study period for the rainfall variable. The study period was defined to make use of the greatest number of stations possible and to guarantee the spatial coverage of the study area, in order to obtain greater reliability in the results.

Of the 160 hydrological stations with precipitation measurement, operated by IDEAM with influence in the department of Boyacá, 134 stations with pluviographic and rainfall readings were selected. When analyzing the information provided by IDEAM, 11 stations were discarded because they did not present a precipitation record within the defined study period, and 15 stations since they did not comply with the Waldwolfowitz and Mann – Whitney tests with a significance level of 1%. as seen in Table 3 and Figure 1.

Table 3 Number of Precipitation Stations that were accepted or rejected during the statistical tests

Season	**Period study**	**Test Waldwolfowitz**	**Test Mann –Whitney**	**Worked**
Accepted	149	141	140	134
Rejected	eleven	8	9	fifteen

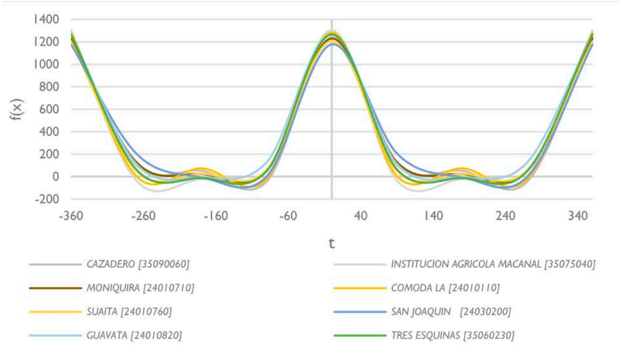

Figure 2 Andrews curves Region No. 10

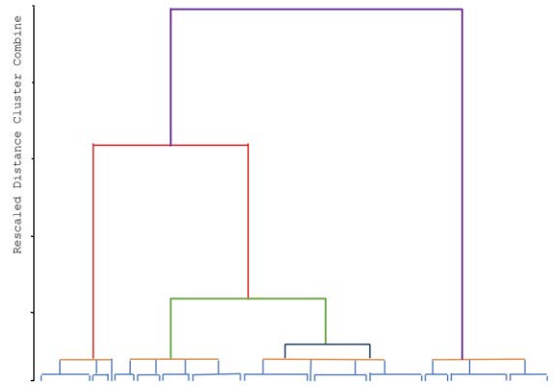

Figure 3 Ward dendrogram for the maximum daily rainfall series

Definition of rainfall regions

From the application of the multivariate statistical method of Andrews Curves, a total of 29 regions were determined, taking into consideration the morphology of the curves and the smallest possible Euclidean distance, since when the variables are closely spaced, a more significant similarity is presented, as seen in Figure 2.

After the application of the Hosking and Wallis tests, as a complementary methodology implemented to verify the definition of the formed groups, it was feasible to observe the acceptance of 17 regions comprised of three or more stations, while 5 do not comply with the corresponding values of H when applying the Andrews method.

The application of the Hosking and Wallis tests to verify the definition of the formed groups through the Andrews method, managed to determine the acceptance of 13 regions composed of three or more sub-basins, while 8 areas did not pass the homogeneity test - H, and 5 were not analyzed as each was composed of less than three stations.

To set the cluster, a dendrogram was generated by each of the methods presented in the methodology, where the gradual grouping of the clusters can be seen, with respect to the Euclidean distance. Figure 3 illustrates, as an example, the Ward Cluster method, for the series of maximum daily Rainfall, of the total stations. The decision criteria for the number of clusters were looking for a number of suitable regions that would allow drawing an imaginary horizontal line at that distance from which considerable jumps are generated.

The Within Groups method for the annual daily maximum Rainfall series, from the total of the stations, yielded a total of 5 regions with an Euclidean cut-off distance of 15, of which 4 regions do not meet the mean of heterogeneity. A station without grouping or atypical occurs, presenting inconsistency. In the Ward Cluster method, for the daily maximum Rainfall series, it allows for the identification of 5 regions, with a cut-off distance of 1.5, of which no region met the mean of heterogeneity.

The discordance indicator made it possible to filter out the highly discordant stations with respect to the identified regions, and thus, it was possible to define the stations that required a data quality analysis and/or their behavior resembled another region or required the generation of a new one.

The precipitation regions emerged by grouping the Thiessen polygons of stations associated with the same region, because of the L-moment analysis, complying with the hypothesis of homogeneous hydrological behavior and assuming that they should produce similar responses, regardless of the neighborhood. Of the seasons, 8 regions were obtained, which can be seen in Figure 4 and Table 4, which meet the critical ranges of heterogeneity for homogeneous regions ($H<2$) and discordance. However, the results showed four stations without grouping as they presented atypical behavior.

It is highlighted that regions 3 and 5 obtained negative H2 and H3 values, characteristics of lower dispersion between the historical series related to the standard dispersion of L-Bias and L-Kurtosis.

During the investigation, uncertainty is identified due to acquisition errors and related data such as: erroneous precipitation readings, failures in the measurement equipment, disturbances due to the environment in which the measuring instrument is located, insufficiency of the rainfall network having considered the orographic variability of the area and randomness in the presence of extreme events in the study area. Similarly, within the systematic process of information analysis, uncertainty is established in the definition of the method for completing missing data, the definition of the study period, continuity,

Table 4 Heterogeneity of Rainfall regions, department of Boyacá

Region	No. Seasons	L-CV	L-SKEW	L-KURT	L-FIVE	H1	H2	H3
1	fifteen	0.11	0.175	0.23	0.099	1.23	1.78	1.32
2	16	0.083	0.055	0.131	0.045	0.85	1.51	1.68
3	28	0.101	0.109	0.181	0.042	1.84	0.33	-0.61
4	twenty-one	0.088	0.038	0.119	0.004	1.65	0.55	1.22
5	23	0.075	0.035	0.151	0.025	0.5	-0.61	-1.12
6	14	0.096	0.046	0.146	0.016	0.36	1.58	1.17
7	7	0.095	0.097	0.118	0.007	1.46	1.46	1.45
8	6	0.109	-0.146	0.157	-0.003	1.75	0.75	0.46

precision, and representativeness of the maximum annual rainfall series.

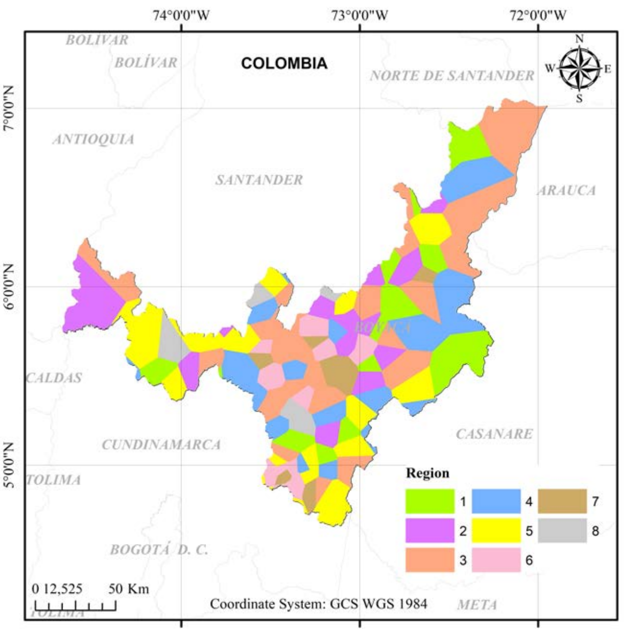

Figure 4 Regions of maximum precipitation in the department of Boyacá

At a global level, the application of statistical techniques such as Cluster, Andrews curve and L-moments have been used in the definition of homogeneous Rainfall regions. In Colombia, considering this type of regionalization, the application of the hierarchical cluster method for the regionalization of monthly precipitation in the Valle del Cauca stands out [30].

For the Cundiboyacense antiplane, a previous study was conducted, where rainfall regions were determined for multi-year dry and humid periods through the Principal Component Analysis method [31], which obtained as a result of homogenization four regions in the dry period and three for the humid season, but only a homogenized region for the area analyzed in the department of Boyacá. In comparison to these results, in the current research, five regions were determined for the same area analyzed in that study. Regarding the maximum annual Rainfall, multiannual Rainfall for dry and humid periods has not been considered.

Another investigation regionalized the patterns of the IDF curves in Boyacá using isolines, a product of the analysis of 20 stations and an error through the cross-validation technique of 10% [32], differing from the present investigation, in which the study was developed with 134 stations and the proposed regions were validated through the L-moments method and discordance tests.

Likewise, it is noteworthy that this type of regionalization is the basis for the formulation of IDF curves in the Department of Boyacá and Colombia, taking into account that the most widely used methodology is the one proposed by Diaz–Granado [33] and that it can also be, as the present case, subject to greater delimitation by region, taking into account sectoral climatic variations, and reducing the associated uncertainties due to the absence of greater discretization.

3. Conclusions

The creation of homogeneous regions is considered a tool to know the hydrological behavior of areas that are not measured or with discontinuities in measurements, allowing the development of projects aimed at the management of water resources.

By combining two multivariate analysis methodologies, such as Andrews and Cluster curves, and regional frequency analysis based on L-moments, a total of 8 homogeneous regions were obtained. This combination is formulated under the assumption that the historical series of the stations have identical distributions, serial independence, and magnitudes vary by a scale factor.

The Cluster and Andrews Curve regionalization methods are susceptible to error, given the subjectivity in defining the clusters.

From these results, it was found that the centroid chaining method is available for the definition of atypical stations, grouping them independently at the beginning of the agglomeration process, which is why it was not taken into account during the definition of the regions.

When comparing the methodologies used, the similarity is found in the station regions with the greatest magnitude in maximum daily Rainfall. Unlike those of smaller magnitude, the Andrews method allows greater discretization.

The regions obtained are characterized by the fact that the stations that compose them are not necessarily adjacent to each other; the regionalization spatially divides the study area into 8 quasi-homogeneous areas, with respect to the historical behavior of the maximum annual precipitation.

4. Declaration of competing interest

We declare that we have no significant competing interests including financial or non-financial, professional, or personal interests interfering with the full and objective presentation of the work described in this manuscript.

5. Funding

This work was supported by the Universidad Santo Tomás Tunja.

6. Author contributions

Laura Garavito designed the methodology, data analysis and helped write the article. Carlos Caro devised the project, analyzed and validated the procedure, and helped write the article. Caterine Casallas provided help with data processing.

7. Data availability statement

The main source of the information analyzed is documentation and records from the IDEAM (Institute of Hydrology, Meteorology and Environmental Studies).

References

[1] M. Escobar, F. Flores-López, and V. Clark. (2011, Nov.) Energy-water-climate planning for development without carbon in latin america and the caribbean. Stockholm Environment Institute. Somerville, MA. [Online]. Available: https://www.sei.org/mediamanager/documents/Publications/Climate/sei-development-without-carbon-ph2.pdf

[2] A. Lujano-Laura, E. Lujano-Laura, and J. P. Quispe-Aragón, "Regionalización de caudales anuales en cuencas del altiplano peruano," *Revista Investigaciones Altoandinas*, vol. 18, no. 2, 2016. [Online]. Available: https://dialnet.unirioja.es/servlet/articulo?codigo=5560552

[3] C. A. Caro-Camargo, O. Pacheco-Merchán, and H. P. Sánchez-Tueros, "Calibration of manning's roughness in non-instrumented rural basins using a distributed hydrological model," *Dyna*, vol. 86, no. 210, Jul. 01, 2019. [Online]. Available: https://doi.org/10.15446/dyna.v86n210.72506

[4] L. M. Albentosa-Sánchez, "La importancia del conocimiento de las fluctuaciones climáticas en los estudios históricos. aproximación al clima de tarragona durante el siglo xviii," *Revista De Geografia, Història I Filosofia*, vol. 4, Mar. 2018. [Online]. Available: https://revistes.urv.cat/index.php/utghf/article/view/1995

[5] C. Iyigun, M. Türkeş, ̇I. Batmaz, C. Yozgatligil, V. Purutçuoğlu, and *et al.*, "Clustering current climate regions of turkey by using a multivariate statistical method," *Theoretical and Applied Climatology*, no. 114, Jan. 03, 2013. [Online]. Available: https://doi.org/10.1007/s00704-012-0823-7

[6] Y. Zhang, S. Moges, and P. Block, "Optimal cluster analysis for objective regionalization of seasonal precipitation in regions of high spatial–temporal variability: Application to western ethiopia," *Journal of Climate*, vol. 29, no. 10, May. 15, 2016. [Online]. Available: https://doi.org/10.1175/JCLI-D-15-0582.1

[7] C. A. Caro1 and E. Bladé, "Water resources management: Green watershed index (gwi)," presented at IOP Conference Series: Earth and Environmental Science, Beijing, China, 2020. [Online]. Available: https://doi.org/10.1088/1755-1315/690/1/012033

[8] A. Ramachandra-Rao and V. V. Srinivas, "Regionalization of watersheds by hybrid-cluster analysis," *Journal of Hydrology*, vol. 2318, no. 1-4, Aug. 18, 2005. [Online]. Available: https://doi.org/10.1016/j.jhydrol.2005.06.003

[9] D. Guzmán, J. F. Ruíz, and M. Cadena. (2014) Regionalización de colombia según la estacionabilidad de la precipitación medial mensual, a través análisis de componentes principales (acp). Subdirección de Meteorología –IDEAM. Colombia. [Online]. Available: https://tinyurl.com/33f2dwjz

[10] L. Pagliero, F. Bouraoui, J. Diels, P. Willems, and N. McIntyre, "Investigating regionalization techniques for large-scale hydrological modelling," *Journal of Hydrology*, vol. 570, Jan. 11, 2019. [Online]. Available: https://doi.org/10.1016/j.jhydrol.2018.12.071

[11] P. Vezza, C. Comoglio, M. Rosso, and A. Viglione, "Low flows regionalization in north-western italy," *Water Resources Management*, vol. 24, May. 06, 2010. [Online]. Available: https://doi.org/10.1007/s11269-010-9647-3

[12] *Guide to Hydrological Practices*, 6th ed., World Technological Organization, Geneva 2, Switzerland, 2009. [Online]. Available: https://library.wmo.int/records/item/36066-guide-to-hydrological-practices-volume-ii

[13] G. Álvarez Olguín, N. Hotait-Salas, and F. Sustaita-Rivera, "Identification of homogeneous hydrological regions through multivariate analysis," *Ingeniería, Investigación y Tecnología*, vol. 12, no. 3, Sep. 2009. [Online]. Available: https://www.scielo.org.mx/pdf/iit/v12n3/v12n3a4.pdf

[14] S. Eslamian. (2014) Handbook of engineering hydrology: modeling, climate change, and variability. C.R. Press. Boca Ratón, FL. [Online]. Available: https://tinyurl.com/bdz9z5hy

[15] C. A. Caro-Camargo and J. A. Bayona-Romero, "Hydro-dynamic modeling for identification of flooding zones in the city of tunja," *Revista Facultad de Ingeniería, Universidad de Antioquia*, vol. 88, Sep. 18, 2018. [Online]. Available: https://doi.org/10.17533/udea.redin.n88a05

[16] N. Dogulu and E. Kentel, "Clustering of hydrological data: a review of methods for runoff predictions in ungauged basins," in *19th EGU General Assembly, EGU2017*, Vienna, Austria, 2017. [Online]. Available: https://ui.adsabs.harvard.edu/abs/2017EGUGA..1912005D/abstract

[17] M. de Oliveira-Cándido and M. Naghettini, "Seaf – a prototype of an expert system for at- site frequency analysis of hydrological annual maxima," *Avances en Recursos Hidráulicos*, vol. 18, Oct. 16, 2008. [Online]. Available: https://www.redalyc.org/pdf/1450/

145012856004.pdf

[18] C. A. Caro-Camargo and L. Gil-Alvarado, "Flood risk management contributions in localized areas. case study: Tunja, colombia," *Tecnología y Ciencias del Agua*, vol. 12, no. 4, Jul. 14, 2021. [Online]. Available: https://doi.org/10.24850/j-tyca-2021-04-09

[19] C. L. Cheng and M. C. Liao, "Regional rainfall level zoning for rainwater harvesting systems in northern taiwan," *Resources, Conservation and Recycling*, vol. 53, no. 8, Apr. 29, 2009. [Online]. Available: https://doi.org/10.1016/j.resconrec.2009.03.006

[20] L. M. Castro-Heredia, Y. Carvajal-Escobar, and □. J. Ávila Díaz, "Análisis clúster como tecnica de análisis exploratorio de registros múltiples en datos meteorológicos," *Revista Escuela de Ingeniería de los Recursos Naturales y del Ambiente EDINAR*, vol. 11, Oct. 08, 2012. [Online]. Available: https://revistaeidenar.univalle.edu.co/revista/ejemplares/11/b.htm

[21] M. Templ, P. Filzmoser, and C. Reimann, "Cluster analysis applied to regional geochemical data: Problems and possibilities," *Applied Geochemistry*, vol. 23, no. 8, Mar. 26, 2008. [Online]. Available: https://doi.org/10.1016/j.apgeochem.2008.03.004

[22] M. Gómez-Suárez, "El análisis cluster en investigación de marketing: Metodología y crítica," *Esic market*, vol. 113, 2002. [Online]. Available: https://tinyurl.com/2fd7ksnu

[23] X. Gong and M. B. Richman, "On the application of cluster analysis to growing season precipitation data in north america east of the rockies," *Journal of Climate*, vol. 8, no. 4, Apr. 01, 1995. [Online]. Available: https://doi.org/10.1175/1520-0442(1995)008<0897:OTAOCA>2.0.CO;2

[24] S. D. la Fuente-Fernández. (2021) Análisis conglomerados. [Ebrary version]. [Online]. Available: https://tinyurl.com/mwx84zy2

[25] J. C. Salazar-Oliveros, "Una metodología para la estimación de curvas de duración de caudales (cdc) en cuencas no instrumentadas. caso de aplicación para colombia en los departamentos de santander y norte de santander," M.S. thesis, Facultad de Ingeniería, Universidad Nacional, Bogotá, Colombia, 2017.

[26] R. Modarres and A. Sarhadi, "Statistically-based regionalization of rainfall climates of iran," *Global and Planetary Change*, vol. 75, no. 1-2, Oct. 28, 2010. [Online]. Available: https://doi.org/10.1016/j.gloplacha.2010.10.009

[27] B. G. H. Hassan and F. Ping, "Regional rainfall frequency analysis for the luanhe basin – by using l-moments and cluster techniques," *APCBEE Procedia*, vol. 1, Jul. 19, 2012. [Online]. Available: https://doi.org/10.1016/j.apcbee.2012.03.021

[28] J. R. M. Hosking, *Regional Frequency Analysis: An Approach Based on L-Moments*. New York: Cambridge University Press, 1997. [Online]. Available: https://doi.org/10.1017/CBO9780511529443

[29] L. Núñez-Galeano and J. D. Giraldo-Osorio, "Adaptation of the l-moments method for the regionalization for maximum annual temperatures in colombia," *Ingenieria y Universidad*, vol. 20, no. 2, Jun. 2016. [Online]. Available: https://doi.org/10.11144/Javeriana.iyu20-2.almr

[30] Y. C. Escobar and J. M. Segura, "Aplicación de métodos estadísticos para la regionalización de precipitación mensual en el valle del cauca," *Meteorología Colombiana*, vol. 5, pp. 13–21, 2002.

[31] D. A. Gómez-Latorre, G. A. Araujo-Carrillo, and Y. Rozo-Leguizamón, "Regionalización de patrones de lluvias para periodos multianuales secos y húmedos en el altiplano cundiboyacense de colombia," *Revista de Climatología*, vol. 22, Dec. 30, 2022. [Online]. Available: https://rclimatol.eu/wp-content/uploads/2023/01/Articulo22l.pdf

[32] J. E. Muñoz-Barragán and E. Zamudio-Huertas, "Regionalización de ecuaciones para el cálculo de curvas de intensidad, duración y frecuencia mediante mapas de isolíneas en el departamento de boyacá," *Tecnura*, vol. 22, no. 58, Dec. 20, 2018. [Online]. Available: https://doi.org/10.14483/22487638.14295

[33] R. Vargas and D. M. Díaz-Granados, "Curvas sintéticas regionalizadas de intensidad-duración-frecuencia para colombia," *Revista de Ingeniería Uniandes*, vol. 1, pp. 1–13, 1998.