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The school bus routing problem: a systematic literature review

El problema de ruteo de buses escolares: una revisión sistemática de literatura

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ABSTRACT: The School Bus Routing Problem (SBRP) is a classic optimization problem with a massive potential for real applications that have a high impact on society. Research interest in this problem is constantly rising. Previous review papers with a time-space between them of 10 years have helped understand the different features studied by the research community about this problem. In this systematic review, we consider two new categories not discussed before: the incorporation of a mixed load composed of multiple schools, along with the inclusion of a smart element related to the availability of user information and communication in real-time to join the smart mobility trend. In addition, we discuss the lack of real applications in the SBRP in university contexts with a focus on the multi-load problems.

RESUMEN: El problema de enrutamiento de autobuses escolares (SBRP) es un problema de optimización clásico que tiene un enorme potencial para aplicaciones reales de alto impacto en la sociedad. El interés de la investigación por este problema aumenta constantemente. Artículos de revisión previos, con un espacio temporal de 10 años entre ellos, han sido de ayuda para comprender las diferentes características estudiadas por la comunidad investigadora sobre este problema. En esta revisión sistemática consideramos dos nuevas categorías no discutidas antes: la incorporación de una carga mixta compuesta por múltiples escuelas, junto con la inclusión de un elemento inteligente, relacionado con la disponibilidad de información del usuario y comunicación en tiempo real. para sumarse a la tendencia de movilidad inteligente. Además, discutimos la falta de aplicaciones reales del SBRP en contextos universitarios con énfasis en los problemas de carga múltiple.

1. Introduction

The School Bus Routing Problem (SBRP) is a real-world problem that impacts not only the school transportation systems but also the mobility in the cities. SPRP involves the design and operation of schedules to provide a transportation service for students from and to schools with effectiveness regarding the satisfaction of the service, efficiency in costs, and safety of the parts involved (operators, students, and schools) while fulfilling several constraints mainly referred to transport the students safely and on-time [1–5].

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As a sign of the impact of this activity, the school bus fleet in the US is more than twice the size of all other forms of mass transit combined [6]. It transports about a third of students to and from school, with an average annual public expense per student approaching US $1,200 in 2018-19 [7]. Even so, technological advances in transport are slowly impacting school transportation operations. For example, only 33% of the American school bus fleet uses GPS to track the buses [6], and 54% of the school transportation operations use routing software.

As a classical optimization problem, many mathematical approaches to solving the SBRP can be found in the literature in the last decade, with variants in the focus of the problems addressed, and the solution methods proposed [2]. In response to these changes, and keeping the classification framework originally proposed by [8], we go further, proposing a distinction of the approaches that take into account the multiple school SBRP problems with mixed loads, the technological and communications advances available to users with more dynamic solutions, and we examine how this classification fits university transport services, that seems to be scarce in the related literature.

As contributions, in this paper, we: (i) Present a clear systematic review process that can be traceable, offering hints on the contribution of the different databases used, and on the trends of the variants or particularities of the problems addressed. (ii) Analyze a specific category of SBRP focused on interuniversity routing, through the consideration of mixed load allowance. (iii) Identify a new dimension in the categorization of SBRP, related to the timeliness of information and response, in terms of static or dynamic contexts. (iv) Analyze how the classical school contexts differ from university contexts, and (v) Identify areas of opportunity for future research.

We have structured this paper as follows: firstly, we offer a concise description of the SBRP. Section 2 provides the primary outcomes of the Systematic Literature Review (SLR), detailing the sub-problems and characteristics addressed in these optimization issues. Moving on to Section 3, we delve into the most prevalent solution approaches. Lastly, Section 4 discusses some of the most pertinent findings and explores the distinctions of this problem within university contexts.

1.1 The school bus routing problem

A typical SBRP begins with the selection of stops (BSS: bus stop selection) and their relationship to the address of residence of the students [9]. In these cases, social factors, and quality of service to the user are important, among which stands out the distance traveled on foot by the student to the stop. The selection of the stops can be solved in several ways: strategies in which first the stops are selected and then the students are assigned to them, called Location – Allocation - Routing (LAR) or vice versa, the Allocation-Routing-Location (ARL) strategies where the students are first grouped and then one stop per group is assigned [8]. The LAR strategies generate solutions with an excessive number of paths because they do not consider the effect of route capacity on the student assignment. In contrast, ARL strategies look for options to overcome this obstacle [10]. Also, LAR can be easily solved as an assignment problem [4] and ARL as a clustering problem [11].

Once the stops have been selected, the routing problem itself (VRP) springs into action, that is, selecting the order of visiting the stops. This sub-problem is denoted BRG: bus route generation. However, to avoid future confusion, the results will be called trips (not routes). Some of the classic variants of VRP apply, such as vehicle capacity, time windows, and consistency [12], which are all known to be Non-Deterministic Polynomial (NP) hard problems. However, several variants of the school context make the SBRP a special and interesting problem. Some of these are the duration of the routes, which is more important from the point of view of quality of service than for cost issues. Here, the riding times are of interest as a service quality indicator. Also, the number of stops, length of stops, and passenger pickup time windows are important. Unlike VRP for logistics, it is not necessarily of interest in maximizing vehicle occupancy. On the contrary, it is necessary to consider the possibility of overcrowding, especially in these post-pandemic times. Additionally, the need to minimize costs in terms of distance or time, or the number of buses and routes, also becomes transcendental when solving SBRP by seeking the optimization of school bus routes [5].

When defined, trips can be sequenced or covered by a fleet of vehicles. This sub-problem is denoted BRS: bus route scheduling. Henceforth, the result of a sequence of stop visits is a *trip,* and the sequence of trips is a *route*. Unlike other contexts, bell times can be fixed and unique (e.g., one time for entrance and one time for school departure), so that the number of trips is usually equivalent to the number of routes since a vehicle cannot serve more of a route on the same day, at least in the same direction. Bus routing and bus scheduling can help improve bus utilization while minimizing the number of routes and the total travel distance [13]. Based on the school system, an SBRP can be categorized into two classes: single school and multiple school. When the context involves multiple schools, multiple schedules, staggered schedules, and mixed loads, then the sequencing problem becomes remarkably interesting. The mixed-load allowance exists as an alternative that provides a better-carrying rate since it allows students to commute to different schools on the same bus route [14]. Due to this reason, there is a tangible similarity between this problem and the pickup and delivery with time windows VRP problem, as both issues involve a set of pick and delivery locations. Operational

Glossary			
General Acronyms		**Acronyms in Model Classification**	
AAC	Artificial ant colony	**C**	Vehicle capacity
ACO	Ant colony optimization	**COL**	Chance of being late
ARL	Allocation – Routing – Location	**COO**	Chance of overcrowding
BirD	bi-objective routing decomposition	**CU**	Capacity utilization
BRG	Bus route generation	**DC**	Demand covering
BRS	Bus route scheduling	**EPT**	Earliest pick-up time
BSS	Bus stop selection	**FB**	Flow balance
CPLEX	Commercial Optimization Software ®	**HO**	Homogeneous
EDA	Estimation distribution algorithm	**HT**	Heterogeneous
GA	Genetic algorithm	**LB**	Load or ride time balance
GAA	Genetic ant algorithm	**MR**	Maximum route length/distance
GPS	Global Positioning System	**MRL**	Maximum route length
GRASP	Greedy randomized adaptative search procedure	**MRT**	Maximum riding time
ILK	Iterated Lin-Kernighan	**MSN**	Minimum student number to create a route
ILP	Integer linear programming	**MSR**	Maximum stops per route
ILS	Hybrid iterated local search	**MWT**	Maximum walking time or distance
IWS	Intelligent water drops algorithm	**N**	Number of buses used
LAR	Location – Allocation – Routing	**NS**	Number of stops
LS	Local search	**NT**	Number of transfers
MIP	Mixed-integer programming	**RB**	Route balance
NP	Non-deterministic polynomial-time	**SBS**	Shared bus stop
SA	Simulated annealing	**SF**	Safety factor
SBA	School bell time adjustment	**STW**	Stop time window
SBRLIB	Library with benchmark instances for SBRP	**SWD**	Total student walking distance
SBRP	School Bus Routing Problem	**TBD**	Total bus travel distance or time
SLR	Systematic literature review	**TC**	Total cost
STP	School bus routing policies	**TCT**	Total commute time
TS	Tabu search	**TRC**	Trip compatibility
VND	Variable neighborhood descent	**TSD**	Total student riding distance or time
VNS	Variable neighborhood search	**TT**	Transfer time
VRP	Vehicle routing problem	**TW**	School time window

practices and research have proved that mixed-load can potentially reduce the need for vehicles as well as the operational cost of the school bus service [15].

Furthermore, there exists the option that a typical input data for these problems, such as the school bell times, becomes a decision variable looking for more efficient solutions. This sub-problem is denoted as SBA: school bell time adjustment. Finally, and similarly to SBA, there is a component that becomes relevant given its ability to optimize specific routing objectives (i.e., total distance traveled when considering a mixed-load policy), which refers to the analysis of school bus routing policies (STP subproblem) [2]. It is worth mentioning that, even though the literature currently defines the STP as a fifth SBRP sub-problem, no publication has been identified as focused on the modeling process of solving this sub-problem (either individually or in conjunction with another SBRP subproblem).

This partition of sub-problems is very similar to how other complex operational problems in transportation are approached, like in airline operations problem, where each sub-problem can be seen analogously to a specific problem: The flight schedule design with the BSS, the allocation of fleets and the routing of airplanes with the BSR, the conformation of pairings with the BSS. The analogy with the BSA is rarely addressed, only in [16]. The complexity of each sub-problem supports the fact that in both applications, the search for more integrated solutions (i.e., simultaneous solutions for two or more sub-problems) is a constant trend in literature [17].

2. Systematic literature review (SLR) methodology

To clarify the selection of the relevant literature revised in this work, we have adopted the Method proposed in [18]. In this section, we describe the first four stages of the SLR. In the first stages, we selected the scope of the search, the sources, the criteria for inclusion and exclusion, and the specific search fields and terms: Table 1 summarizes this process. In the first stages, we applied a conservative criterion: the paper was included when in doubt, and the decision was made after the complete paper was read. Decisions on acceptance of a paper were made in pairs. When there was doubt, a third (senior) researcher took the final decision.

The process yielded a total of 336 initial results across all databases. The first filter was to eliminate repeated papers among databases, resulting in 205 papers. Then, eighty-three papers were selected based on the reading

Table 1 Scope of the Search

	Source scope
Database selection	Scopus, ProQuest, IEEE and Web of Science
Source type	Articles in journals & Conference proceedings
Fields	Scopus: Title and Title-Abs-Key || ProQuest: Document Title, Abstract and Keywords/identifiers || IEEE: All Metadata || Web of Science: Topic (Title, Abstract, Author Keywords, and Keywords Plus)
Filters	Language: English || Year: >2010
	Inclusion/exclusion criteria
Boolean phrase	**Scopus and ProQuest:** ("smart rout*" OR "urban rout*" OR "bus rout*") AND (simulat* OR select* OR optimiz* OR estimat* OR model*) AND (university OR college OR school) AND (bus)) **IEEE Xplore:** "All Metadata": smart NEAR/3 rout* OR urban NEAR/3 rout* AND "All Metadata": model* OR simul* OR estimat* OR select* AND "All Metadata": university OR college OR school AND "All Metadata": bus NEAR/3 rout* **Web of Science:** ((((TS=("smart rout*" or "urban rout*" or "bus rout*")) AND TS=((simulat* or select* or optimiz* or estimat* or model*))) AND TS=((university or college or school))) AND TS=(bus))
Exclusion criteria	Papers not in English that passed the filter. Papers dealing with urban services (not related to school bus transport) Papers dealing only with tracking systems without mathematical optimization approach Papers focused on the bus design. Papers dealing with social topics (disabilities, security, among others) Papers dealing with a rural service environment. Papers dealing with single route problems. Papers with no access by the research team.
Other inclusion Criteria	Mendeley suggestions Backward citations

of their title and abstract. We excluded eight due to access restrictions. Finally, after backward inclusions, a total of 49 papers were selected as the corpus of this review. Figure 1 shows the source of the selected corpus. The number "1" in the orange circle inside the gray one in Figure 1 denotes that one paper, reference [19] was selected from ProQuest and Web of Science simultaneously. The proportions by databases are Scopus 76%, Web of Science 59%, ProQuest 16%, IEEE Xplore 12%. However, only one paper was solely found in ProQuest, so this database contributed little to the final corpus. Interestingly, most of the papers were found exclusively in only one of the databases. This justifies the practice of not restricting the search to a single database from the beginning of the review process.

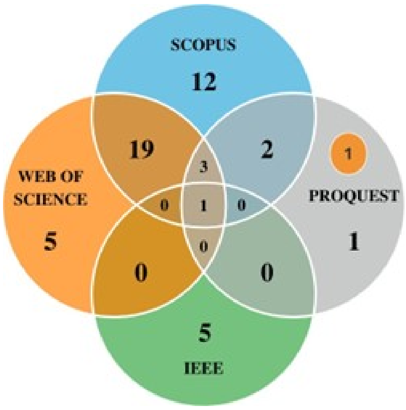


Figure 1 Venn Diagram of the Corpus

2.1 SBRP Sub-problems

As previously mentioned, the different SBRPs in literature are classified into four "sub-problems" that can be addressed either independently, sequentially, or simultaneously: bus stop selection (BSS), bus route generation (BRG), bus route scheduling (BRS), and school bell time adjustment (SBA). Though strategic transportation policy (STP) was initially considered, no publication addressing it was found. Table 2 summarizes the key features of each sub-problem and identifies the publications dealing with each one. Figure 2a. shows the yearly frequency of publications in the corpus, classified for each SBR sub-problem, showing the steady interest in the last six years. This figure supports that BRG is the dominant sub-problem of interest, with 80% of the papers addressing it, while SBA is only considered by four publications.

The Venn diagram in Figure 3 shows that SBR sub-problems are considered together. Nevertheless, it was found that the most frequently solved problem in the corpus was BRG with 35%. Additionally, 22% of the studies consider the combination BSS – BRG, and 2% deal with three sub-problems at the same time (BRS + BRG + BRS). Only two studies deal with the four sub-problems. The number "1" in the blue circle inside the green one in Figure 3 means that one paper studies BRG and SBA simultaneously [20]. BSS is mostly addressed together with BRG, since in a scheduling problem, one key factor is the compatibility of trips, as it can help reduce the overall number of buses. The trips themselves are an input for the scheduling problem, so it is essential to provide a solution with a set of highly compatible trips so it can result in routes of multiple trips served by one bus.

2.2 Characteristics of the SBRP considered in literature

A classification scheme based on the problem characteristics was proposed in [8] and later followed and complemented in [2]. In addition to the sub-problem type, it considers six categories, as shown in 0. To facilitate the reading, we have adopted the same convention. To complement this classification, we have added a category that involves the dynamic aspects of the problem together with the availability of real-time communications with the passenger.

Figure 4a shows that approaches dealing with homogeneous fleets prevail, assuming the papers that do not specify the fleet type normally model homogeneous fleets. The few papers that consider heterogeneous fleets integrate two or three sub-problems at the same time [21]. Additionally, the papers that consider heterogeneous fleets are those that simultaneously model multiple school SBRP, due to the mixed loads. Recalling that the papers in the corpus are in an urban socio-geographical context, Figure 4b shows major interest in China and the US. In addition, Figure 4c shows that almost two-thirds of the papers focus on working by multi-objective functions.

Finally, as shown in Figure 4d, 16% of papers consider the dynamic aspect of the problem, by using GPS systems and IT to receive data from the students and react dynamically to the changes (e.g., [1, 25, 32, 50]). This evidences that most of the papers focus on a static response with historical data or even test data to validate models, and very few use real-time information to propose dynamic approaches. It is worth noting that studies regarding dynamic aspects of the SBRP but focused on bus design, mobile applications, and tracking systems of passengers of school bus services rather than optimization problems were excluded, as shown in Table 1.

Regarding objective functions, total bus travel time or distance (TBD) stands out with 30 out of 49 papers (i.e., [3, 46]), followed by the total cost (TC) with 27 papers (i.e. [29, 38, 41]), and the number of buses used (N) with 18 papers (i.e., [15, 45]. The total riding and the walking distance are followed by 6 papers each. The least common are the number of shared stops, trip compatibility, maximum route length, and the number of transfers, as well as route balance and total commute time. Furthermore, 35% of the studies optimize more than one objective function, although the practical need to consider multi-objective problems has been recognized [23]. Among these studies, in [43], the multi-objective function included the minimization of the route balance, seeking to make every school bus travel the same distance as possible, and to minimizing the total number of school buses and the total travel distance. Besides, in [11], the minimization of the total commute time of students, involving walking, riding, and service time was added to the minimization of the number of stops. Finally, in [9], the minimizing straphangers (i.e., a standing passenger in a bus) were used in the multi-objective function along with total travel time.

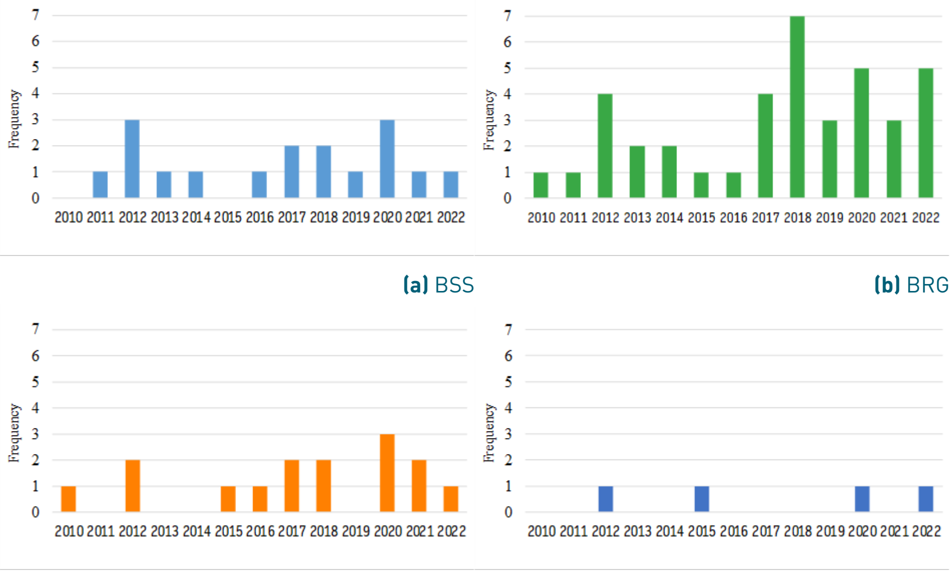
Regarding the constraints, as seen in Table 3 , in addition to the standard constraints that are specific to each subproblem, the most common conditions considered in school bus contexts are school time windows at the start and end of the route, flow balance, which stable connection within the routing, the maximum length/distance of the route for upper limit allowed, demand coverage, the maximum walking distance, earliest pick-up time for the period in which the vehicle is allowed to arrive at the stop before the indicated time, maximum walking time, minimum number of students to create a route, among others.

Additional limitations related to the well-being of the student are the maximum driving time, service time (loading/unloading), transfer time, walking distances/speeds, overcrowding, straphangers meaning the standing passengers in the vehicle, the possibility of arriving late, the maximum number of stops per route and load tracking.

Finally, regarding the new classification category, most of the papers assume a static condition, explained by the assumption that students, their locations, and the time bells will remain constant during the school period. Contexts-based and solutions supported on real-time data are classified as dynamic.

3. Methods to solve the SBRP

The SBRP is a type of Vehicle Routing Problem (VRP) involving multiple variants, which further increase the complexity of the problem. To understand the way the SBRPs are solved by the corpus, Table 4 categorizes the papers based on their primary general solution approach in three options: exact, heuristic, and metaheuristic methods. The first one encompasses papers focused on practical models without necessarily introducing new solution methods but utilizing commercial optimization solvers. They can also introduce combinations or improvements of well-known exact methods with specific heuristics. The other two groups give special attention



(a) BSS **(b)** BRG

(c) BRS **(d)** SBA

Figure 2 Annual frequency of publications in SBRP sub-problems

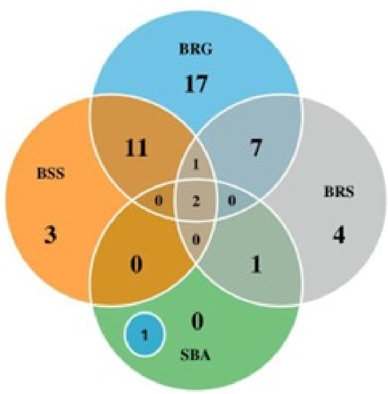


Figure 3 Venn diagram. Studies on multiple sub-problemss

to the solution methods, proving improvements in computational time or showing their capability of solving larger problems. Papers that blend two metaheuristics are labeled as a hybrid, while those incorporating additional heuristic methods within a metaheuristic framework are categorized under the respective metaheuristic classification.

3.1 Exact methods

The different SBRP sub-problems are formulated as either integer linear programming (ILP) or mixed integer linear (MIP) models that are difficult to solve (i.e., NP-hard). These models have shown satisfactory performance in tackling practical but not large-scale problems for schools. Exact methods are suitable for practical-sized cases with acceptable times, for instances with less than 50 nodes [1]. Nonetheless, the use of exact methods has areas of opportunity in instances with sizes adapted to reality, since the method does not facilitate decision-making in reasonable times, especially in cases where mixed-load strategies are considered [29]. The revised literature proposing variants in the formulations as well as in the classical exact methods are all focused on increasing the efficiency in the computational effort –or time– and lowering the gaps to optimality.

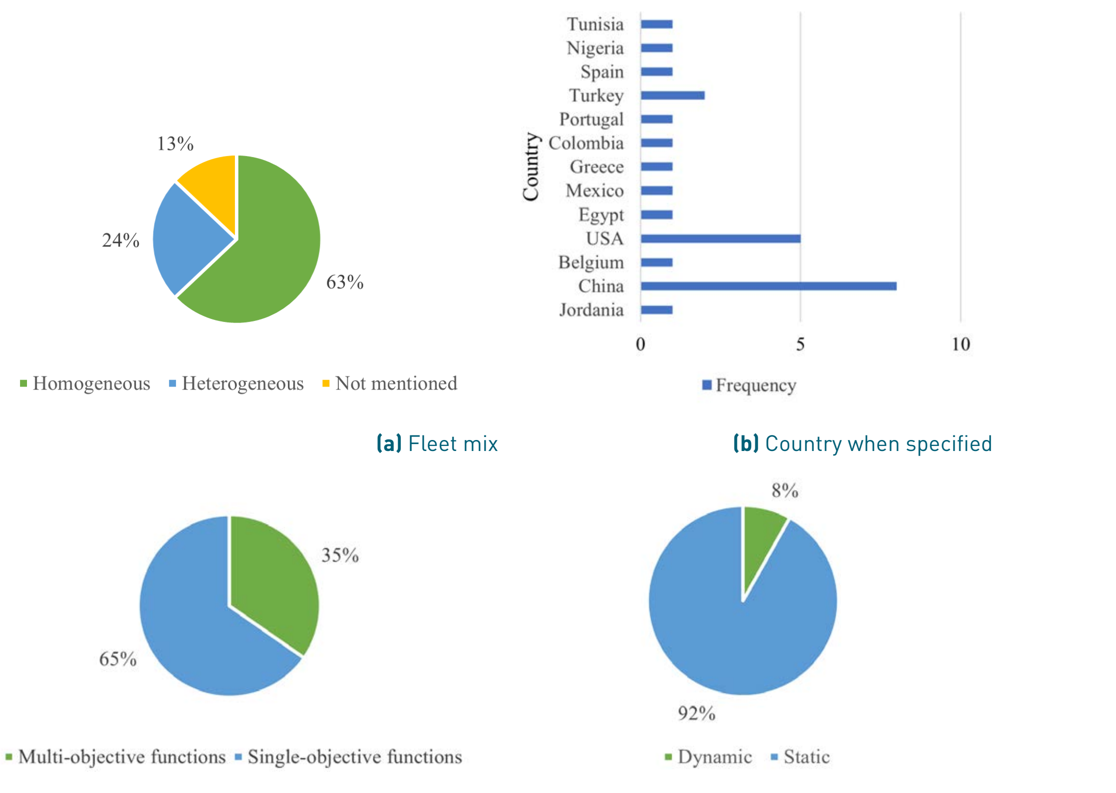
A school bus service in Lisbon was designed using SBRP formulation [27]. The solution applied consisted of a MIP divided into two steps: BSS + BRG. It proved to be able to obtain solutions for medium-sized and single-school problems. Furthermore, an example of implementation in a real context showed that certain changes had to be made to the initial solution. These changes increased the estimated travel time; however, the solutions became more acceptable for users considering bus stops and congested areas.

Later, in [17], an exact branch and piece algorithm with an emphasis on efficiency issues of the column generation of a covering formulation was developed and proved to solve many SBRP instances to optimality, where some of the largest instances solved contain 40 stops and 800 students. The application of several techniques, including the use of the exact and heuristic pricing algorithms, bounding procedures, a column pool manager, and stabilization techniques, as well as their rigid branching approach for the branch-and-price tree was reported to increase the performance of the column

Table 2 School Bus Routing Sub-problems

Sub-problem	Objective	Input data	Comments	References (*)
BSS Bus stop selection	To identify a bus stop location for each student.	Potential stops Student location or zonal demand Time constraints	Approaches: First stops selection, then students' assignment (location- allocation). First students grouping, then assign to a stop simultaneously	[1, 3, 10, 11, 17, 19, 21–24] [25–33]
BRG Bus route generation	To build the school bus routes. Includes the generation generation of a single trip, a series of trips (routes) for a single school, or routes for multiple schools.	Stops location. Demand per location Fleet features (size, capacity, cost, etc.) Operation policies (max length, max time, mixed loading, etc.)	The most frequent problem. When both BRG and BRS are studied, the concept of trip is distinguished: BRG deals with trips, while BRS deals with routes. Trip: begins with an empty bus at an origin point, visits the stops (where students get into/get off the bus), and finishes at the school. Route: series of trips covered by the same bus.	[1, 3, 5, 9–11, 14, 15, 17, 19] [20–23, 25, 26, 28–30] [31, 34–40] [41–49] [50–53]
BRS Bus route scheduling	To identify the timetable for buses Trips durations, Fleet	Time windows Load/unload times pick-up, drivers' brakes. features Distance matrices	Social quality measures: early	[1, 4, 11, 13, 21, 29, 31, 35, 47] [49, 50, 52, 54, 55]
SBA School Bell Time Adjustment	To adjust school schedules with bus schedules.	Several school's time windows. Fleet features operational policies	Usually solved in conjunction with other sub-problems.	[1, 20, 35, 50]

(*) references are repeated if the same paper covers more than one sub-problem.



(a) Fleet mix

(b) Country when specified

(c) Function type

(d) Response timing

Figure 4 Corpus Composition

generation. In [56], a new formulation for the Mixed Capacitated General Routing Problem and a two-phased branch and cut algorithm to solve it exactly were proposed, in which the new branch and cut constructed an initial solution through an effective location-based heuristic. Optimality on two benchmark sets was proved through the algorithm, and a group of larger instances was studied for the first time, gaining a remaining gap to optimality below a threshold of 20%.

In [57], a bi-objective routing decomposition (BirD) algorithm allowed to solve a mixed integer optimization

Table 3 Classification categories of SBRP based on problem characteristics

Category	Alternatives
Number of schools	Single Multiple
Service environment	Urban Rural Both
Response Timing (New)	Static Dynamic
Load type	Yes: Mixed-load Allowed No: Mixed-load Not allowed
Fleet mix	HO: Homogeneous HT: Heterogeneous
Objectives	N: Number of buses used * TBD: Total bus travel distance or time TSD: Total student riding distance or time SWD: Total student walking distance* RB: Route Balance MRL: Maximum route length LB: Load or ride time balance SBS: Shared bus stop CU: Capacity utilization* TC: Total cost* TRC: Trip compatibility SF: Safety factor NT: Number of transfers NS: Number of stops TCT: Total Commute Time DC: Demand covering*
Constraints	C: Vehicle capacity MRT: Maximum riding time TW: School time window MWT: Maximum walking time or distance EPT: Earliest pick-up time MSN: Minimum student number to create a route TT: Transfer time** STW: Stop time window MSR: Maximum stops per route COO: Chance of overcrowding COL: Chance of being late MR: Maximum route length/distance FB: Flow balance
Further considerations	Service time (load/unloading) Walking distance/speed Overcrowding / Straphangers Load tracking

Based on [2, 8]. * Used also as a constraint, ** Used also as an objective

model for the school bell time selection problem, which is a generalization of the quadratic assignment problem that aims to evaluate transportation costs when each school is assigned a particular bell time. The BirD algorithm was able to improve by 20% of computational cost besides considering the best option for each school and multiple routes. Finally, exact solutions for an environmentally related VRP considering maximum route length, vehicle capacity, and fleet type were provided in [42]. A flow-based mixed integer linear program was formulated to compare heterogeneous and homogeneous fleets. The heterogeneous fleet solution minimized 30% of the total cost and enabled the utilization of all the buses' capacity.

Stochastic approaches are also used with exact methods. In [55], numerical experimentation was performed with instances with zero transition time, non-zero transition time, and multiple scenarios. Transition times refer to transit time between routes, which are assumed to be significantly smaller than route times. In [48], a robust optimal schedule times model, which considers the stochastic nature of the problem, was suggested to obtain an exact solution for the cost of delays and idle times. A small-scale computational instance was considered and solved through the mathematical programming solver named CPLEX to verify the integrity of the model, which concludes stochastic and time-dependent transportation networks must be considered more in the assumptions of the SBRP.

3.2 Heuristics methods

In [31], a school-decomposition heuristic algorithm is proposed for solving the routing and scheduling problem. The objective consisted of maximizing trip compatibility while potentially minimizing the number of buses. To test the performance of the algorithm, it was compared with traditional routing by solving eight midsize problems, which showed that the proposed model could reduce the number of buses needed by up to 25%. Following the topic, an insertion-based Minimum Cost Matching with a Post Improvement algorithm was presented in [47], which is a two-step heuristic adopting the trip compatibility idea presented to solve the multi-SBRP under single load assumptions. The first step found an initial solution using the iterative minimum cost matching-based insertion heuristic, and then, the initial trips were improved using a Simulated Annealing (SA) and Tabu Search (TS) hybrid method. Experiments based on benchmark problems [4] were conducted and showed that the proposed heuristic algorithm could save up to 25% of buses. This strategy has proven to be effective and could be used to solve similar VRP with time windows and trip compatibility, as the heuristic algorithms could find solutions with higher quality than the exact methods in a much shorter time, as well as in large-size problems.

Additionally, in [29], a heuristic method was followed to improve computational times and to find near-optimal solutions to the problem. The algorithm started with a constructive approach known as cluster first-route second. It ended with a machine-scheduling heuristic to determine the order and time intervals in which routes needed to be executed. They showed that a mixed-load strategy produces a better use of resources, by reducing the number of buses by 25% in comparison to the single-load strategy (the latter being solved by an exact

Table 4 Table 4 Solution Methods for the SBRP

Approach	Methods	References
Exact	Decomposition (BirD) Stochastics	[57]
	Integer Linear Programming ILP	[34]
	Mixed Integer Programming MIP	[4, 13, 27, 29, 33, 42, 55, 56, 58]
	Column Generation [17]	
Heuristic	Minimum Cost Matching with Post-Improvement Algorithm	[47]
	Machine-scheduling	[29]
	School decomposition algorithm	[31]
	Mixed-load Improvement Algorithm	[45]
	Augmented Lagrangian Relaxation	[14]
Metaheuristic	Evolutionary Genetic Algorithm (GA)	[2, 19, 26, 32, 35, 38, 43, 44]
	Ant Colony Optimization (ACO)	[11, 44, 52]
	Estimation Distribution Algorithm (EDA)	[19]
	Intelligent Water Drops Algorithm (IWD)	[3]
	Local Search Tabu Search (TS)	[1, 15, 25]
	Hybrid iterated local search (ILS)	[23, 39, 51]
	(Iterated) Local Search (LS)	[22, 41]
	Simulated Annealing (SA)	[13, 47]
Constructive	Greedy Randomized Adaptative Search Procedure (GRASP)	[50]
Hybrid	GA + TS	[46]
	GAA: GA + ACO	[49]
	GA + Exact +TS and SA+Exact+TS	[28]
	GA+ILS	[30]
	ILS + SP	[40]
	TS+VNS	[54]
	AAC+VNLS	[24]
	GRASP + VND	[10]
	ACO+ ILK	[36]

method). A mixed-load improvement algorithm was proposed in [45] with modifications of a single load plan, such as the introduction of a simple relocation operator that functions for single and multiple schools. The model focused on finding the optimal routes while reducing the number of vehicles required. Computational experiments were performed to generate benchmarks and real-world instances. For instance, the proposed algorithm was applied to 7 real-world instances with data collected from school districts in the US, resulting in a decrease of 22% of the current buses.

Furthermore, an Augmented Lagrangian Relaxation was applied in [14] to simplify the base Lagrangian model into a 0-1 quadratic programming model with balanced linear flow constraints. After decomposition and linearization, this augmented model can be further partitioned into different linear multiple-product sub-problems. Therefore, a cyclic-block coordinate-descent method is suggested to iteratively solve linear multi-commodity sub-problems based on a multiple school SBRP with mixed-load allowance and a heterogeneous fleet considering pickup-time windows and school-bell-ring constraints.

The use of heuristic methods to solve the School Bus Routing Problem has proved to be more efficient than exact methods in terms of computational time and quality of solutions. This solution method is useful for large-scale problems because it has been shown to give better results for real-world problems when compared. The tendency shows the development of heuristic methods to solve problems with mixed-load, multiple schools, and heterogeneous fleet characteristics as it, according to the authors mentioned, provides a more significant outcome; however, there is an opportunity for further research.

3.3 Metaheuristics methods

Mixed-load conditions in an SBRP are known to add complexity to the models and solution methods used. In [44] a multi-objective Ant Colony Optimization (ACO) algorithm integrated a routing heuristic algorithm. Comparing mixed-load and single-load situations, results showed a greater reduction in the number of buses when using mixed-load formulation. A two-stage metaheuristic algorithm was developed in [52] to solve an SBRP with a mixed-load plan, in which schools share resources for the delivery of students. The combination of the aggregation-based clustering algorithm with an improved ant colony optimization solved this problem with virtual stops and interscholastic transportation models. The results on built instances showed that the second one is better for large-scale cases and the first one works better for small-scale instances but with larger running times.

Deepening into evolutionary algorithms, [43] proposed a Genetic Algorithm (GA) to solve a multi-objective

SBRP with the optimization objectives of route balance, total number of school buses, and total travel distance. To achieve this, the authors proposed an improved non-dominated sorting GA that can outperform the standard NSGA-II and the Multi-Objective Evolutionary Algorithm due to the stability of the algorithm, by presenting a solution that has the best balance degree (in terms of the route selection). Similarly, a GA that uses operators (mutation-S) to solve SBRP is used in [35]. As a result, the algorithm can achieve good results by optimizing the total bus travel distance and the number of buses to be used within a route which was tested on a set of instances of the SBRPLIB library.

Combinations of metaheuristic algorithms with clustering techniques are also used to solve the SBRP. A two-stage solution method is proposed in [11] that considers, in stage I, an iterative clustering method based on the k-means and density-based spatial clustering of applications with noise, and in stage II, the ACO. The results show that the school bus stop location and routing problem with walking accessibility and mixed-load provide convenient and safe school bus services in dense areas, but in sparse ones, a door-to-door school bus service is better. Regarding the algorithm, the authors concluded that the proposed ACO is robust and less vulnerable, although some parameters must be adjusted to achieve a better solution for sparse areas.

On the other hand, an Intelligent Water Drops Algorithm is applied in [3], which is a metaheuristic swarm-based optimization technique. The proposed algorithm results in obtaining reasonable SBRP solution results in good computational times. Another algorithm based on distribution estimation EDA has been proposed to improve the traditional sequential selection-routing approach, by obtaining different and better results than with a GA, as well as adding an alternative to solve permutation-based representation problems, in this particular case, with applications related to logistics [19].

Regarding local search (LS) approaches, in [41], a metaheuristic composed of an ILS and a set partitioning procedure (SP) was introduced specifically for heterogeneous school bus routing problems. [39] proposed a metaheuristic framework in which LS and neighborhood operators are used to simplify the design and implementation in SBRP, based on several scenarios such as single and multiple schools, mixed-load allowance, and heterogeneous or homogeneous fleets. Also, in [1] and [15], SBRPs were solved with a tailored TS and a pick-up and delivery problem with a time windows approach with a record-to-record travel metaheuristic for mixed-load problems. Both improve computational times while giving quality solutions.

A partial allocation LS algorithm discussed in [22] is a metaheuristic approach based on three main steps: selecting the bus stops, constructing the solution by computing routes and allocating students, and applying several LS procedures. The results showed that it is an extremely competitive method in terms of closeness to solution and computing times. Also, a metaheuristic with online learning to solve a multi-school heterogeneous-fleet school bus routing problem was presented in [51]. The online learning mechanism was integrated with the VNS heuristic in the iterated local search (ILS) framework. This algorithm was effective for solving mixed-load (and single-load) problems and shows its ability to increase solution quality together with the speed of solution convergence. On the other hand, insertion-based heuristics, and greedy randomized adaptive search procedures (GRASP) were used to minimize the number of buses and times with good results in terms of bell time adjustments, service, punctuality, and shorter student ride time [50].

In [47], a two-step heuristic approach is applied to solve the SBRP under single-load assumptions, considering a method improved by using a Simulated Annealing (SA) and Tabu Search. The results of this approach showed that the two-step proposed heuristic improves single-load problem solutions regarding benchmark problems.

Hybrid approaches

In the last decade, a trend for combining the strengths of different metaheuristics in the same approach has been observed. For example, in [24], a hybrid metaheuristic based on the Artificial Ant Colony (AAC) with a VNS to plan the bus routing problem is proposed, which leads to an improvement in the performance of the AAC. Their proposal was applied in a Tunisian case, showing that the solution produced by their metaheuristic is highly dependent on the choice of the LS and that the solutions were competitive and consistent when compared with the results of the bus network provided.

Previously, in [49], a Genetic Ant Algorithm (GAA) was proposed, that introduces the evolutionary process of genetic algorithm to improve the optimizing efficiency of the ant colony algorithm. This combination improves the optimization performance and boosts convergence, enabling the algorithm to combine randomicity and determinacy at the same time. Another hybrid approach was stated in [10], which is integrated by a construction phase in which a GRASP is used, followed by a VND improvement phase. Both phases are executed sequentially in an iterative way to find the best solution much faster giving the possibility to manage larger scale instances. However, there is still an area of opportunity to expand the approach to be able to involve more aspects such as multiple schools and testing in real scenarios. Also, a GIS-based framework along with clustering techniques, network cutting techniques, and a hybrid ACO with the iterated Lin-Kernighan (ILK) local improvement heuristic is proposed in [36] for solving the SBRP as a split delivery vehicle routing problem.

Most recently, in [30], a memetic algorithm for a heterogeneous fleet school bus routing problem is proposed. This type of algorithm is considered a hybrid genetic algorithm (GA) because it involves LS procedures. Comparing the memetic algorithm with a greedy and genetic algorithm, the computational time was much greater even though it has high consistency in obtained solutions. Additionally, hybrid iterated LS and set portioning procedure (ILS + SP) metaheuristic algorithms were used for solving SBRP with multiple planning scenarios such as single and mixed-load, and homogeneous and heterogeneous fleets in large-scale multi-school SBRP [40]. The results proved the efficiency of the developed algorithm by outperforming the existing SBRP for multiple-school cases.

Finally, a hybrid metaheuristic method is developed in [54], which combines TS and the VNS. The study concludes that this search procedure, which is numerically compared with the exact algorithm (via CPLEX) and the TS metaheuristic on a small-scale network, can generate superior routes and paths, while still making efficient use of available vehicles.

Bi-level Approaches

Bi- and multi-level approaches involve a hierarchical organization to distribute and streamline decision-making. Specifically, in the bi-level, there are two levels and in each of them, part of the decision variables is controlled [28]. A bi-level mathematical model was constructed to predict students' response when designing an efficient transportation system by applying two hybrid metaheuristic approaches (GA-EX-TS and SA-EX-TS) on a location-allocation routing strategy, in the first one using GA to locate bus station, EX to refer to an exact method to allocate the students and TS for the routing, and in the second approach using SA instead of GA to locate the stations, both resulting appropriate for solving large scale problems in less computational time than with exact methods, without any of them showing dominance in their performance [28].

Similarly, another bi-level bi-objective location routing was proposed to minimize routing costs and maximize the profit of the transportation company based on leader-follower games [46]. In this case, a hybrid method (i.e., ILP and Metaheuristics) was developed to solve this problem. Computational comparison between explicit enumeration and hybrid methods was along with demand and cost analysis.

4. Discussion

SBRP research mainly focuses on single-school problems, although it is based on a multi-school system. Adding multiple and heterogeneous features adds complexity to the models and implementations in real contexts of a single school at a time are simpler [27]. Recent research has indicated the study of mixed-load allowance on route as an alternative approach to the multiple school SBRP, known as the mixed-load SBRP [15] (See Figure 5). Henceforth, the development of algorithms to solve this type of problem is increasing the attention of literature as well as the inclusion of heterogeneous fleets, which increases the complexity of the problem [39, 40]. However, heterogeneous fleet solutions have demonstrated improved results involving total costs and seat occupation on SBRPs [42].

Time-dependence of travel times was rarely considered in SBRPs. Only a couple of studies explicitly addressed this issue. In the first one, a robust optimization model that minimizes the worst-case total cost is solved in a stochastic time-dependent network for a single-school configuration and homogeneous fleet. The cost is a function of delays and the disutility of travel times [48]. In [54], the approach considers path flexibility between nodes to be included in the route. It combines TS and VNS to solve the model and evaluate the effects of congestion on cost, travel time, and distance. In both studies, a small-scale instance was used to validate the proposals. On the other side, time dependency is implicitly considered in the dynamic approaches that will be discussed next.

The consideration of multi-objective versions of SBRP models has also gained interest in the literature. Henceforth, the elements of these objective functions that appear most in the literature refer to minimizing total cost, number of buses, travel distance, or travel times. Nonetheless, some authors begin to integrate elements that consider the minimization of environmental impact into multi-objective functions [17, 29]. Environmental concerns have been addressed, following the research trends of the variant of the VRP called the Green Vehicle Routing Problem, such as the Environment-Friendly School Bus Routing Problem [42]. Inefficient school bus route management can result in more energy usage and pollution than necessary; meanwhile, the opposite has a positive impact on the environment and students' health [26]. A minimum number of documents oriented to dynamic environments, with real-time data considerations, can be observed during the review [32].

Interest persists in the development of heuristic, metaheuristic, and exact solution methods. A concentration of work on metaheuristic methods is evident, due to the NP-hard complexity of the problem. Even so, the development of new and improved solution approaches remains a potential for further research. Another trendy use in solution methods is linking data mining techniques as grouping methods, especially in stop selection [28, 33, 46].

4.1 Dynamic approaches

We introduced a novel category, labeled "response timing" in Table 3, referring to the temporal aspects of input data used in the optimization model. Static works predominantly constitute the corpus, involving pre-given input data. On the other hand, dynamic studies have only appeared in the last decade, as shown in Figure 5. They entail the real-time acquisition of input data, facilitated by the easy and inexpensive tracking capacity of users, owing to advances in GPSs, traffic sensors, and mobile communication systems such as mobility as a service (MaaS) applications and cellphones. When transport solutions integrate this data-driven approach, they are recognized as "smart solutions" [59]. These IT innovations invite us to identify up-to-date bus routes during the daily operation. Smart and dynamic approaches mark a burgeoning trend that has garnered attention in the last decade [1, 32, 60].

Examples of this approach include GPS, which can be used to identify real-time student waiting locations and the school bus online, as in [32, 60, 61] to generate dynamic adjustments on the route to last-minute changes such as non-attending students. Data is sent to a server where a metaheuristic method (e.g., GA or ACO) runs to find the dynamic route, and finally, IT tools are used to visualize the solution. Connected to mobility or traffic applications, these approaches inherently account for time-dependent variations in speed. These approaches, when connected to mobility or traffic applications implicitly regard the time-dependent variations in speed. On the other hand, cellphone applications are used to make a reservation for a seat on the bus, and the dynamic route can be simplified by skipping the stops where there is no demand for them, therefore lowering travel times, as in [1].

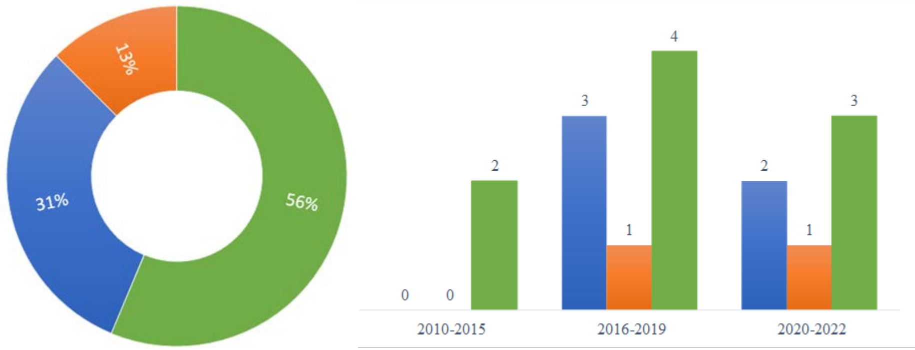
4.2 University contexts

In the vast majority, the applications with real data were developed for school cases, leaving aside the application in universities, except for only two works [1, 29]. These institutions have important challenges such as the heterogeneity of schedules and, in most contexts, mixed loads, higher dispersion of users, multiple depots of the same institution, and the absence of the restriction of 100% demand covering, unlike an SBRP in a school context. In [1], where a university context is considered, one can seek to optimize social interests such as demand coverage and travel time under limited resources. Another relevant characteristic refers to the consideration of mixed loads in university contexts. For this, the use of a mixed-load strategy with students from different schools sharing buses allows for minimizing operational costs and optimally satisfying time windows [29]. Both works highlight the similarity of these problems to public transport, regarding the need to comply with requirements such as the arrival reliability of multiple bell times, the consequent multiple arrivals and departures, and other aspects such as traffic congestion, mobility problems, high operational costs, and high transfer times. We conclude that, even though some authors have joined efforts to contribute to the solution of the SBRP in university contexts, as shown in Figure 5, the effort that the literature has directed towards this matter is still scarce but promising.

4.3 Mixed-load allowance

According to the literature, there are two types of routing in multi-school problems: single-load route and mixed-load route. In a single-load route, the bus is only allowed to pick up students from the assigned school at each stop, meaning the students have the same destination school. Meanwhile, in a mixed-load route, the students on a bus may have different destination schools [2]. As stated in [45], the SBRP should be considered a mixed-load plan because the essence of the problem is aimed at impacting several school districts. Since students from suburban areas are often dispersed around the area, prohibiting mixed-loading could be inefficient. Therefore, the transfer school bus operation plan must allow mixed loads. In these cases, certain characteristics, such as the time windows of the schools, are key, so much so that being able to adjust the time windows can improve the effect of allowing mixed load [11].

Researchers are currently studying the SBRP with a mixed-load route because of the high operation cost and low utilization of buses the schools are facing. A mixed-load plan can consider interscholastic transportation, which refers to picking up students from different schools and delivering them to their respective schools. A single-load plan was compared to a mixed-load plan to share resources between multiple schools to improve the utilization of buses [52]. In [8], it is pointed out that considering a single-load plan can be too restrictive, resulting in an excessive number of buses being required



(a) a) Proportion (16 papers in total)

(b) b) Time distribution

Figure 5 Distribution of papers dealing with the discussion subtopics

for passengers. Therefore, allowing mixed loading can achieve greater flexibility and cost savings. In addition, it has been shown that the multiple-school SBRP could be modeled as a continuous approximation model, or as a kind of pickup and delivery VRP with a homogeneous fleet where student stops are considered as pickup locations and schools as delivery ones [14].

Mixed load SBRP has been successfully addressed with heuristic and metaheuristic algorithms. They have proved to produce better use of resources, such as shorter routes, less operating costs, greener operation, as well as fewer buses in rush hours, less vehicle congestion, and reduced student travel times. Nevertheless, for large-scale mixed SBRPs, enhancing the performance of the actual algorithms is necessary [29, 31]. Lastly, in [29], load and mixed-load strategies are used to optimize the use of buses in the school transportation system of Bogota, concluding that interuniversity routing, through the consideration of mixed-load allowance, generates a better use of resources and lower operation costs.

5. Conclusions and recommendations for future work

A differentiating element of the work presented here is providing a systematic study of the literature detailing each stage of the process at a level higher than that shown in [8] to foster greater reliability when replicating or developing a future SLR on SBRP.

Although a significant amount of research has been conducted regarding SBRP in the last few decades, especially between the gap of the reviews in [2, 8], promising opportunities remain regarding (i) the multiple SBRP with a mixed load, (ii) university contexts, and (iii) the dynamic contexts with real-time data exist, due to the increasing interest and the few previous publications in these specific contexts.

- **(i)** The consideration of multiple SBRP modeling has attracted the attention of the literature for the last decades. However, there is a tangible opportunity to solve this problem in multi-school systems and university contexts, given the variants involved, such as heterogeneous students, heterogeneous fleet, multiple time windows, and mixed load.
- **(ii)** There is an area of opportunity in the consideration of the dynamic aspect when modeling and solving SBRP, especially from the university context. Access to different mobile and technological resources or applications makes the development of dynamic routing solutions possible. However, very few studies have considered real-time data when modeling dynamic SBRP. Henceforth, focusing the research on the use of systems or applications such as GPS, AI, or IT to receive information from university students and react in real-time for routing and scheduling solutions is a potential future direction.
- **(iii)** Finally, it is worth exploring multi-university or multi-site solutions as they can help address the complexity and potential for efficiency increases. The solution of the SBRP from the interuniversity context, considering heterogeneous fleets, mixed loads, and real-time data, through the modeling of multi-objective functions and the construction of metaheuristic solution algorithms can contribute in a tangible way to the literature in the area, through the development of sustainable, effective, and efficient school routing systems.

Declaration of competing interest

We declare that we have no significant competing interests including financial or non-financial, professional, or personal interests interfering with the full and objective presentation of the work described in this manuscript.

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Data availability statement

The authors confirm that the methodology to select the corpus is described in the article, and the data supporting the findings of this study are available within the article and references. Working tables of the corpus can be supplied upon request from the corresponding author.

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