Revista Facultad de Ingeniería

UNIVERSIDAD DE ANTIOQUIA 1803

Title: **Resilience index for earth retaining systems in road infrastructure**

Authors: Jorge Arturo Pineda, Sherley Catheryne Larrañaga-Rubio and Mario Guadalupe González-Pérez

DOI: **10.17533/udea.redin.20250369**

To appear in: *Revista Facultad de Ingeniería Universidad de Antioquia*

Received: March 07, 2024
Accepted: March 28, 2025
Available Online: March 28, 2025

Please cite this article as: J. A. Pineda, S. C. Larrañaga-Rubio and M. G. González-Pérez. Resilience index for earth retaining systems in road infrastructure, *Revista Facultad de Ingeniería Universidad de Antioquia*. [Online]. Available: https://www.doi.org/10.17533/udea.redin.202500369

Resilience index for earth retaining systems in road infrastructure

Índice de resiliencia para sistemas de contención de tierras en carreteras

Jorge Arturo Pineda[1] https://orcid.org/0000-0002-0953-9745, Sherley Catheryne Larrañaga-Rubio[2] https://orcid.org/0000-0002-5457-5948, Mario Guadalupe González-Pérez[3*] https://orcid.org/0000-0002-5457-5948

[1]Facultad Tecnológica, Universidad Distrital Francisco José de Caldas. Calle 68D Bis A Sur # 49F - 70, Localidad de Ciudad Bolivar, Bogotá, Cundinamarca, Colombia.

[2]Escuela Tecnica Superior de Ingeniería Civil, Universidad Politécnica de Madrid. C. Ramiro de Maeztu, 7, Moncloa - Aravaca, 28040. Madrid, España.

[3]Centro Universitario de Tonalá, Universidad de Guadalajara. Nuevo Perif. Ote. 555, Ejido San José, Tateposco. 45425 Tonalá. Jalisco, México.

*Corresponding author: Mario Guadalupe González Pérez

E-mail: mario.gperez@academicos.udg.mx

KEYWORDS

Entropy; resilience; sustainability; flexible retaining system; rigid retaining system

Entropía; resiliencia; sostenibilidad; sistema de contención flexible; sistema de contención rígido

ABSTRACT: The construction of roads often involves cuts and embankments, requiring the use of earth retaining walls. This study estimates a resilience index for these systems based on four fundamental criteria: robustness, redundancy, resourcefulness, and recovery. To achieve this, quantitative weightings based on a multicriteria analysis were applied to the service life of both a rigid and a flexible retaining wall systems near the city of Bogotá, Colombia, considering maximum surface acceleration and groundwater level variations. The results indicate a resilience index of 0.78 for the rigid system and 0.82 for the flexible system, indicating that the flexible system exhibits a higher resilience capacity. These findings can inform risk management policies and resource optimization strategies, ultimately reducing entropy during the design, construction, and operation phases of road projects.

RESUMEN: La construcción de carreteras implica muchas veces cortes y terraplenes que hacen necesario implementar muros de contención de tierras. Este estudio estima un índice de resiliencia para estos sistemas a partir de cuatro criterios básicos: robustez, redundancia, capacidad de gestión y recuperación rápidas. Para ello, se aplicaron ponderaciones cuantitativas en la vida útil de un sistema de contención rígido y otro flexible en la Ciudad de Bogotá, Colombia, combinando la aceleración máxima en la superficie y variaciones en el nivel freático. El resultado arrojó, un índice de 0.78 para el sistema rígido y 0.82 para el sistema flexible, lo que sugiere que el sistema flexible cuenta con una mayor capacidad de resiliencia. Estos hallazgos pueden ayudar a definir políticas de gestión del riesgo y

optimización de recursos, con la intención de reducir la entropía en las fases de diseño, construcción y operación de proyectos carreteros.

1. Introduction

1.1 Theorical backround

In recent years, it has been evident that factors such as urban population growth, climate change, the environmental impact of infrastructure projects, the occurrence of natural and/or anthropogenic disasters, and socio-economic inequalities significantly impact the established physical environment. Therefore, the concept of resilience has gained prominence, becoming one of the cornerstones for the sustainability of engineering systems [1-3]. Within this context, numerous researchers have sought to link the lifecycle of civil engineering projects with the concept of resilience [4-8], which has previously been addressed from various engineering disciplines [9, 10],

Resilience is understood not only from technical and functional perspectives but also from economic, social, and environmental aspects [2, 11]. Nevertheless, its interpretation is closely linked to the specific field of study in which it is applied. The concept was first introduced in ecological research in the 1970s by [12], who proposed that ecosystems self-sustaining and persisting despite disturbances and changes. In other words, the conditions of a complex, non-equilibrium, and unstable system evolve simultaneously, leading to a reorganized and resilient state [13, 35].

In the field of engineering, the concept of resilience was introduced by [14], who defined it as “the time it takes for a system to return to a state prior to the disturbance [5], or the speed at which a variable displaced from equilibrium returns to it (return time)”; in other words, how quickly a system reverts to the conditions prior to the disturbance. However, from other perspectives, reversibility is nothing more than a mere idealization since every real process is inherently irreversible, and reversibility can only be discussed in special cases [15]. Hence, this concept is closely related to the generation of intrasystemic entropy.

Entropy has been employed within various theoretical and conceptual frameworks, which have not always been convergent. Its origins trace back to thermodynamics [16], specifically stemming from the publication in 1824 of the work *Réflexions sur la puissance motrice du feu et sur les machines propres à développer cette puissance* by [17]. Subsequently, in 1865, Rudolf Clausius introduced the concept of entropy, formally articulating the second law of thermodynamics governing irreversible processes (known as the Law of Entropy). However, it was years later when the Austrian-born mathematician Ludwig Boltzmann facilitated the understanding of the term through the famous equation $S = K \log W$, where entropy (S) represents the logarithm of the number of microstates corresponding to a macrostate [18]. Even the physicist Erwin Schrödinger mentioned, “Let me recall that entropy is a direct measure of molecular disorder, viz. its logarithm” [19]. Indeed, the concept has been essential in laying the groundwork for Claude E. Shannon and Warren Weaver's mathematical theory of communication and Ludwig Von Bertalanffy's general systems theory [20, 21]. Nevertheless, “...there is no general interrelation between the information entropy and the thermodynamical entropy... the information entropy is a subjective, measurement-system-dependent feature, while the thermodynamical entropy is an objective property of the physical system" [22].

Hence, entropy and resilience maintain a close relationship as they both pertain to the properties of a system. While one relates to the disorder that this system undergoes over time due to intra-systemic and extra-systemic causes, the other identifies the system's capacity to reverse the generated disorder. Therefore, the concept of resilience emphasizes the need among professionals to establish practical methods for quantifying it, ensuring it is not merely understood as a theoretical concept [23, 24]. This is especially important when the system can experience varying degrees of impact based on its intrinsic properties and the magnitude of experienced entropy [25, 26].

The model proposed for assessing resilience must, first and foremost, specify its configuration and the perturbations of interest. That is, it should define the state of the system under consideration and the perturbations of interest. Specifically in engineering, resilience pertains to a system's preparedness and response to disruptive events [11, 25]. It is primarily associated with the system's intrinsic abilities to mitigate the effects of these events efficiently and effectively through proper resource organization, as well as the design and implementation of strategies to address factors that affect system stability [27, 35, 37].

Based on the above, resilience has become a fundamental element in modern approaches to the design, management, construction, and maintenance of infrastructure projects, from their inception to the end of their service life. This is especially crucial in an era where sustainability has become a key driver in decision-making processes related to the future of both anthropogenic and ecosystem processes. Ecosystems provide the necessary materials and energy for the existence and functioning of anthropogenic processes, but they also receive the waste generated by these processes, leading to entropy. Therefore, guidelines have been established with a view to achieving sustainable development goals, as "Civil engineers often find themselves at the center of the development-environment conflict, especially in their roles as planners and builders" [28, 36].

2. Methodology

In this research, two commonly used earth retention systems in road construction were analyzed on five criteria. Metrics for obtaining resilience were proposed based on the following considerations:

a) Robustness was represented by the ultimate limit state (ULS) and the serviceability limit state (SLS).
b) Recovery was characterized by recovery in limit states over time.
c) Redundancy was quantified by the number of replaceable components within the geotechnical infrastructure network, calculated based on its entropy.
d) Resourcefulness was represented by construction costs, maintenance, mitigation, and repair of the structure concerning an available budget [6].
e) Impacts were quantified based on technical, environmental, economic, and social perspectives from specialists.
f) On-site inspections were conducted to gather information in the study area.

Regarding item e), considering the viewpoints of specialists, the methodology used by Kermanshachi et al., was followed as it has proven effective in reducing uncertainty [29]. This method or research technique commonly referred to as *Delphi* instrumentation was designed in the early 1960s by Norman Dalkey and Olaf Hermes to establish consensus on relevant topics. The expert's judgment becomes a reliable source, and in some cases, the only one available to forecast the future through opinion comparison [30].

In this context, an expert is considered as “both the individual and a group of individuals or organizations capable of offering conclusive assessments of a given problem and making recommendations regarding its essential moments with a maximum level of competence” [31]. Indeed, subjectivity is a fundamental part of this exercise since it lacks initial mathematical foundation and is more based on the subject's experience. However, the resulting information can be quantifiable and subjected to a logical and rigorous analysis since it aims to reduce the interquartile range between Q1 and Q3, approaching a forced convergence of Q2 [30, 32, 33]. It is also important to note that there is no specific rule regarding the number of experts to participate since the method does not seek a substantial but qualified participation. However, some suggest that an interval between seven and thirty experts is acceptable [33, 34].

Based on these considerations, a panel of 8 interdisciplinary experts was formed to assign numerical values with the intention of evaluating the aforementioned six criteria. The methodological procedure followed the following steps:

i) An expert judgment was carried out to determine the importance of each criterion.
ii) Scores were assigned to each criterion based on its importance.
iii) Weights were assigned to each component of resilience based on their scores.

To define the weights of the resilience components, the panel of experts was composed of civil engineering professionals with expertise in geotechnics, risk management, and sustainability, including professionals in economics and public management. They met periodically to discuss the characteristics of each component and their relative importance. In the discussion, experts considered the following factors:

w) The importance of each component for the safety and functionality of the retention system.
ww) The difficulty of assessing each component.
www) The availability of data to assess each component.

In this regard, the panel awarded scores to each criterion on a scale of 0 to 1, with 0 being the lowest importance and 1 being the highest importance. In summary, the qualitative-quantitative methodology used allowed for the assignment of these weights to the resilience components to ensure consistent weighting with the knowledge and experience of professionals in each sector.

2.1 Resilience Index Estimation Process in Earth Retention Systems

Furthermore, in this work, we based the performance equation proposed by Holling [12], incorporating a different approach that determines the relationship between the performance level before and after an interruption, as well as the internal factors that the equation generally relates [35, 37]. As presented in the previous section, the resilience index (RI) represents the ability of an earth retention system to recover after a disruptive event within a certain time interval. However, due to the limitations in simulating the time of a disruptive event and the time it takes for the system to fully regain its functionality, we propose **Equation 1** to estimate the resilience index. This equation relates the resilience at an initial period $R\ (t_1)$, which is the moment when the structure is functioning normally, and the resilience at a final period $R\ (t_2)$. This final period refers to the moment after a disruptive event has occurred, and the structure's normal conditions have completely changed. Therefore, considering the scale on which resilience has been measured by different authors [6, 10], we use an index evaluation range from 0 to 1, where 0 represents a lack of resilience, and 1 represents full recovery.

$$\mathrm{RI} = \mathrm{R}(\mathrm{t}_1) / \mathrm{R}(\mathrm{t}_2) \tag{1}$$

Based on the above equation, **Equation 2** and **Equation 3** quantitatively represent the resilience of the system at the two analyzed moments (t_1 and t_2), considering the influence of the four criteria (a-d) mentioned earlier according to [35].

$$\mathrm{R}(\mathrm{t}_1) = 0.35\mathrm{Rb}_1 + 0.35\mathrm{Rd}_1 + 0.20\mathrm{Rs}_1 + 0.1\mathrm{Re}_1 \tag{2}$$

$$\mathrm{R}(\mathrm{t}_2) = 0.15\mathrm{Rb}_2 + 0.20\mathrm{Rd}_2 + 0.30\mathrm{Rs}_2 + 0.35\mathrm{Re}_2 \tag{3}$$

In this equation, the terms $Rb1$, $Rd1$, $Rs1$, and $Re1$ represent robustness, redundancy, management capacity, and rapid recovery at time $t1$. Similarly, $Rb2$, $Rd2$, $Rs2$, and $Re2$ denote the same criteria at time $t2$. The numerical factors in each equation correspond to weightings that either amplify or diminish the influence of each criterion. It is important to emphasize that this characteristic depends not only on the physical and mechanical conditions of a specific earth retention structure but also on environmental, demographic, social, economic, and technical factors inherent to the behavior and functionality of the structure, all of which form part of a broader earth retention system. The following section outlines the determination of resilience criteria, considering that both quantitative and qualitative aspects of earth retention systems are evaluated for its estimation. The weighting factors used in the resilience index assessment of the earth retention system presented here were derived from a retrospective study of cases involving earth retention walls with compromised stability in the central region of Colombia, affecting both roads and urban environments [7][35][36][37].

2.2 Robustness determination criteria (Rb)

This criterion assesses how secure the earth retention system is in its configuration and degree of functionality. It allows us to evaluate the system's capacity to withstand disruptive events to minimize their impact on the structure and its surroundings. Therefore, nine aspects are evaluated to characterize each system. These aspects include both qualitative and quantitative factors. Qualitative aspects include the evaluation of the structure's inclination or displacement, the presence of drainage works in and around the structure, the presence of vegetation, and the condition of the pavement to identify mass movements. Additional quantitative aspects include factors such as compressive strength, damage due to surface fissures or cracks, damage to the support mesh of the structure, and damage due to moisture. A relationship was established between the surface area of the structure and the affected area based on the specific conditions for each type of structure. To quantify this criterion, **Equation 4** and **Equation 5** were used. Equation 4 represents the evaluation of rigid earth retention systems, while equation 5 represents the evaluation of flexible earth retention systems.

$$\boldsymbol{Rb(t_n) = 0.20Rb_1 + 0.15Rb_2 + 0.10Rb_3 + 0.05Rb_4 + 0.10Rb_5 + 0.20Rb_6 + 0.15Rb_7 + 0.05Rb_8} \quad (4)$$

$$Rb\,(t_n) = 0.25Rb_1 + 0.15Rb_2 + 0.10Rb_3 + 0.10Rb_4 + 0.10Rb_5 + 0.30Rb_9 \quad (5)$$

The aspects analyzed for each type of earth retention system are treated as a unit, and their evaluation varies depending on the time being assessed. However, their weighting in the previous equations remains constant over time because their evaluation directly impacts the index. The assessment of each aspect represented by Rb is established according to the proposal described in **Table 1**, and the assigned value for each aspect is determined by its relevance at the time being analyzed. The weightings assigned in the measurement of criteria for resilience index analysis $R(t_1)$ are based on the approaches presented by [8]. For example, in the case of urban infrastructure systems, robustness (Rb_1) can be estimated through the analysis of design conditions, considering the function and service of each component. In contrast, Rb_2 is determined by analyzing the service conditions of the components of the infrastructure following an event that has caused a service interruption. The resulting capacity of each component should enable it to continue functioning in the face of future atypical events within the system.

Table 1 Ratings for Aspects of the Robustness Criterion (Rb)

Aspect	*Description*	*Value*
	Vertical structure, without representative displacements, and fulfilling the function of providing stability to the slope.	1.0

Rb_1: Inclination or displacement in the structure	Vertical structure and/or with some degree of inclination and/or minor or non-representative displacements. It fulfills its role in providing stability to the slope.	0.7
	Structure with a significant degree of inclination and/or displacements: Satisfactory	0.4
	Structure with a significant degree of inclination and/or displacements. Fails to provide stability to the slope.	0.0
Rb_2: Drainage works within the retaining structure	It includes drainage structures such as pipes, box culverts, geodrains, or others. It is observed that they fulfill their functionality.	1.0
	It includes drainage structures such as pipes, box culverts, geocomposite drains, or others. It is observed that the majority of these structures fulfill their function.	0.7
	It has few drainage structures, showing inadequate water drainage through other areas, and/or it fails to direct water through all drainage systems.	0.4
	There is no water drainage from the structures, or they are obstructed by vegetation, and there are no drainage facilities in place.	0.0
Rb_3: Artworks near the structure	Presence of drainage works such as box culverts, ditches, sewers, wells, and other infrastructure, providing adequate drainage services.	1.0
	Presence of drainage works such as box culverts, ditches, sewers, wells, and other structures indicates that most of the existing ones are functioning as intended.	0.7
	Presence of drainage works such as box culverts, ditches, sewers, wells, and other structures, and/or it lacks sufficient structures and/or a significant portion of these structures do not fulfill their functionality.	0.4
	It lacks appropriate drainage works and/or lacks maintenance, and/or there is no evidence of the presence of these structures, and/or they do not fulfill their functionality.	0.0
Rb_4: Vegetation on the Ground	The presence of vegetation on the slope is evident, and it is vertical.	1.0
	The presence of vegetation on the slope is observed, and/or a slight degree of inclination and/or inactive mass removal movements are detected.	0.7
	Low presence of vegetation on the slope and/or moderate slope angle and/or presence of active mass movement.	0.4
	Without vegetative cover on the slope, and/or a high degree of inclination, and/or active mass movement.	0.0
Rb_5: Road condition	The pavement is in good condition and does not exhibit any pathologies such as cracks, fissures, alligator cracking, depressions, or bench detachment (loss of bench), and it fulfills its functionality.	1.0
	Pavement with minor issues that do not hinder its functionality.	0.7
	Pavement with mild and/or moderate pathologies does not impede its functionality, but it may lead to more significant issues in the future.	0.4
	Pavement with severe pathologies that hinder the functioning of the road and affect the surrounding areas, including populations and supply.	0.0

Rb_6: Compressive strength (Cs) of the concrete structure	Cs ≥ 35 Mpa 35 MPa > Cs ≥ 32 MPa 32 MPa > Cs ≥ 28 MPa 28 MPa > Cs	1.0 0.7 0.3 0.0
Rb_7: Cracks and concrete structure fractures (% affected area)	0% 0- 5% 5 - 10% 10 - 30% > 30%	1.00 0.75 0.50 0.25 0.00
Rb_8: Moisture in the concrete structure (% of affected area)	0- 10% 10 - 20% 20 - 40% > 50%	1.00 0.70 0.35 0.00
Rb_9: State of the mesh in the gabion structure (% affected area)	0% 0 - 10% 10 - 20% 20 - 40% > 40%	1.00 0.75 0.50 0.25 0.00

Source: Authors, adapted from [35]

2.3 Determination of the Redundancy Criterion (Rd)

This criterion represents how structurally secure the system is, and it consists of two fundamental aspects: the reliability index and the safety factor. To determine it, a reliability analysis was conducted using slope stability software, which performs probabilistic analysis through the *Monte Carlo* method with a sample size of 1000. Cohesion and friction angle of each material, as well as the standard deviation and minimum and maximum relative values of each variable, were defined as random variables [35]. This analysis was carried out for four conditions:

i) Normal groundwater level without seismic activity
ii) Elevated groundwater level without seismic activity
iii) Normal groundwater level with seismic activity
iv) Elevated groundwater level with seismic activity (most critical case).

To quantify this criterion, **Equation 6** and **Equation 7** were used, taking into account the two moments when the system's resilience is being evaluated, namely t_1 and t_2. Here, the aspects within redundancy are linearly related, with the weighting factor for the reliability index having a greater magnitude than the safety factor at the first moment, given its importance just before the disruptive event. Conversely, when the disruptive event occurs, the safety factor becomes more relevant due to potential ground instability.

$$Rd\ (t_1) = 0.6Rd_1 + 0.4Rd_2 \qquad (6)$$

$$Rd\ (t_2) = 0.4Rd_1 + 0.6Rd_2 \quad (7)$$

Where Rd_1 and Rd_2 refer to the reliability index and the safety factor respectively, in the two moments analyzed (t_1 and t_2). For the evaluation of the safety factor, it was established that the higher it is, the higher its rating should be. For its part, the reliability index was determined according to the performance of the structure [35] [36] [37], which indicates that the higher this is, the lower the probability of failure and the greater the reliability of the structure. **Table 2** presents the proposed assessment of each aspect.

Table 2 Redundancy Values (Rd)

Aspect	Description	Value
Rd1: Security Factor (F.S.)	F.S. ≥ 2,0	1.00
	2 > F.S. ≥ 1.5	0.70
	1.5 > F.S. >1	0.50
	F.S. = 1	0.20
	F.S. < 1	0.00
Rd2: Reliability Index	High	1.00
	Good	0.85
	Over the mean	0.70
	Under the mean	0.50
	Poor	0.30
	Insatisfactoy	0.10
	Dangerous	0.00

Source: Authors, adapted from [35]

2.4 Determination of the Resourcefulness Criterion

Considering that this criterion is related to the management activities carried out on the earth retaining system, such as maintenance, monitoring, recovery, or reconstruction, six aspects are evaluated, including budget, the economic importance of the road, vegetation, drainage works, road type, and population. These variables help assess the maintenance of retaining systems and the management of resources and actions taken to mitigate disaster risk. For the quantification of this criterion, **Equation 8** is proposed because the analyzed structures are located on the same section of the road under study, making the management and maintenance performed on them similar (**Table 3**).

$$\boldsymbol{Rs\ (tn) = 0.20Rs_1 + 0.25Rs_2 + 0.10Rs_3 + 0.15Rs_4 + 0.10Rs_5 + 0.20Rs_6} \quad (8)$$

Table 3 Ratings for Aspects of the Management Capacity Criterion (Rs)

Aspect	Description	Value
Rs_1: Table of Rating for Aspects of the Management Capacity Criterion Based on Economic Activities	Activities that are essential for the supply, manufacturing, transportation, and commercialization of goods, as well as the transportation of the high-impact population.	1.0
	Activities related to the supply, manufacturing, transportation, and commercialization of goods, with a medium level of impact on the population, are carried out.	0.8
	Activities related to the supply, manufacturing, transportation, and/or commercialization of goods, as well as the transportation of the population with low impact.	0.5
	Activities related to basic population communication and transportation are carried out.	0.3
Rs_2: Budget for risk management.	The area has policies for constant monitoring of potential risks, and these policies are up to date.	1.0
	The policies for monitoring potential risks are deficient and/or outdated.	0.4
	There is not any follow up	0.0
Rs_3: The state of vegetation in terms of maintenance.	The policies for monitoring potential risks are deficient and/or outdated.	1.0
Z	The area surrounding the structures exhibits a moderate condition.	0.5
	In the area adjacent to the structures, there is a lack of cleaning and pruning.	0.0
Rs_4: Drainage works maintenance	They are free of obstructions and clean.	1.00
	It has minor obstructions.	0.70
	Displays moderate obstructions	0.40
	It is completely obstructed due to lack of maintenance.	0.0
Rs_5: Surrounding population	Población > 60.000	1.0
	12.501 - 60.000	0.8
	2.501 - 12.500	0.5
	0 - 2.500	0.3
Rs_6: Road type	Primary	1.0
	Secondary	0.7
	Terciary	0.4

Source: Authors, adapted from [35]

2.5 Determination for the Fast Recovery Criterion

Within the field of engineering, this concept refers to the actions to be taken in response to a disruptive event that compromises the functionality of the earth retention system [7][8][35]. This criterion considers factors such as the distance to material sources (e.g., nearby quarries) for the reconstruction of the structures, as well as the social and economic impact on affected populations and their commercial or productive activities. It is important to note that while recovery in terms of fixed costs [4], [6], [13] has been evaluated in previous studies, no economic values or fixed costs are considered in this case. These variables change over time and can introduce a degree of uncertainty when included in the analysis. For the quantification of this criterion, a linear equation (Equation 9) is proposed, which remains the same for both time points, $t1$ and $t2$, when the resilience of the systems is evaluated.

$$\boldsymbol{Re(t_n) = 0.40Re_1 + 0.30Re_2 + 0.30Re_3} \quad (9)$$

Where $Re(t_n)$ is the fast recovery criterion evaluated at a known time, tn, which is the moment when the system's condition is being assessed, i.e., the time before the disruptive event and the time after its occurrence. Re_1, Re_2 y Re_3 refer to the aspects that are considered for recovery, and these are presented in **Table 4**, which also provides the numerical ratings for each aspect based on whether the system meets the description provided for each aspect. Finally, the numerical values accompanying the expressions Re_1, Re_2 y Re_3 are the weighting values based on the importance of each aspect within the criterion.

Table 4 Ratings for Aspects in the Fast Recovery Criterion (Re)

Aspect	Description	Value
Re_1: Minimum distance to quarries for recovery	Distance ≤ 5 km	1
	5 km < Distance ≤ 10 km	0.75
	10 km < Distance ≤ 20 km	0.5
	20 km < Distance ≤ 30 km	0.25
	Distance > 30 km	0.00
Re_2: Social impact in terms of recovery	These activities are crucial for the supply, manufacturing, transportation, and commercialization of goods, as well as for the transportation of the population, having a high impact.	1.00
	Activities related to the supply, manufacturing, transportation, and marketing of goods, as well as the transportation of the moderately affected population, are carried out.	0.75
	Activities related to supply, manufacturing, transportation, trade of goods, and low-impact population transportation.	0.5

	Basic population communication and transportation activities are carried out.	0.25
Re_3: Economic impact in terms of recovery	The area has current policies related to prompt reconstruction in case of emergencies.	1
	The reconstruction monitoring policies are deficient and/or outdated.	0.75
	There are no policies associated with the prompt reconstruction in case of an eventuality or emergency.	0.25

Source: Authors, adapted from [35]

3. Case of study

3.1 Estimation of the Resilience Index in Earth Retaining Systems near Bogotá, Colombia

Following the previous sections, two earth retaining systems were analyzed, using some data from [7]. The first system uses a rigid retaining structure (concrete), while the second system utilizes a flexible retaining structure (stone-based gabions). Both systems are located on a roadway in the central area of Bogotá, Colombia (**Figure 1**).

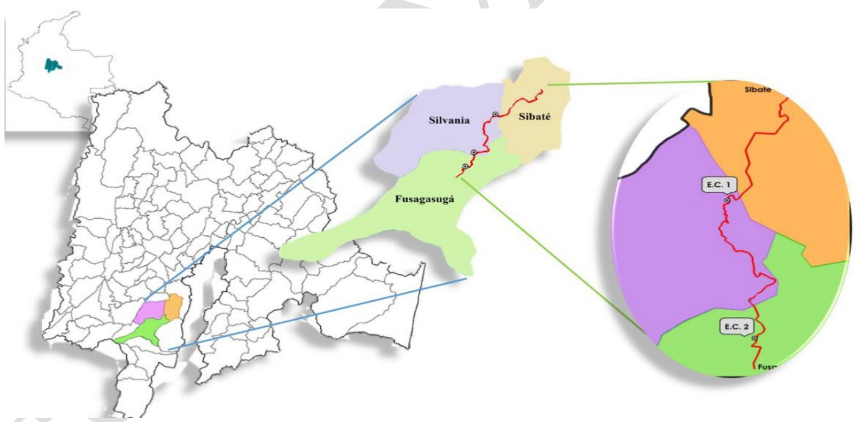

Figure 1 Location of the two analyzed earth retaining systems.

The area in which the analyzed earth retaining systems are located is characterized by geological instability, as it experiences various types of mass movement. Additionally, throughout the year, there are significant rainfall events that act as triggers for soil instability. These systems are situated in mountainous and undulating terrains within an active geological fault. Therefore, a topographic study was conducted, involving a survey conducted by a drone flying at an average altitude of 60 meters, to generate a digital elevation model (DEM). The cross-sectional profiles of the terrain contain the extracted structures.

Based on this information, a probabilistic stability analysis model was developed [7]. This model required extracting four cross-sectional profiles from each structure because the terrain is irregular, and stability varies depending on the topography and the section of the structure (**Figure 2**).

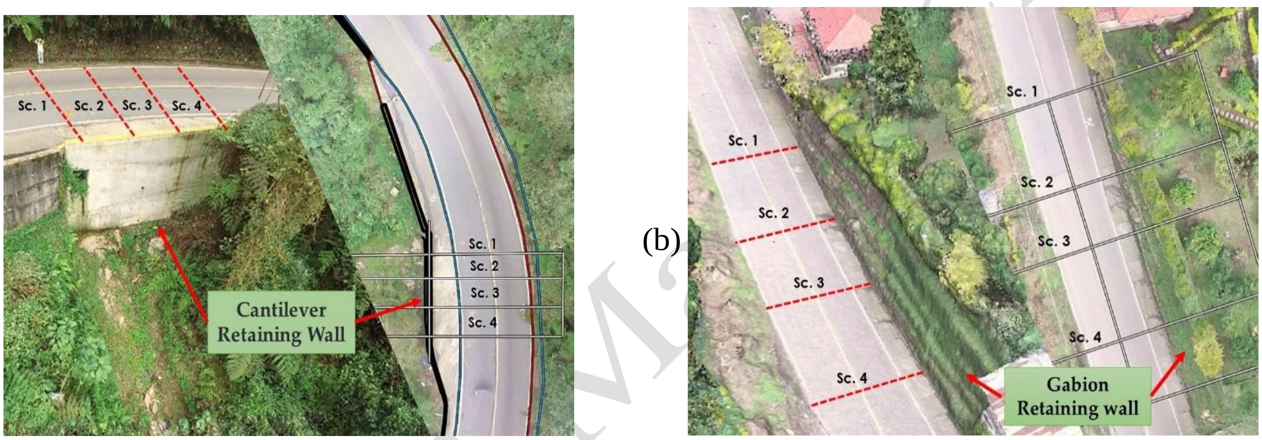

Figure 2 DME rigid system (a) y flexible system (b). Source: Authors

Subsequently, the stratigraphic profile of each system was defined, obtained from Colombian government agencies. The average stratigraphic profile of the rigid earth retaining system contains Type I and Type II materials. The first is a colluvial deposit consisting of sandstone gravel in a silty sandy matrix, with gray and brown colors, classified in the Unified Soil Classification System (USCS) as GM. Material II is an intercalation of moderately weathered dark gray to black mudstone and sandstone with an RQD value of 11% because the rock quality is very poor. In the case of the flexible earth retaining system, the average stratigraphic profile contains Type III and Type IV materials. Type III material is slightly silty Havana sand with medium to fine gravel and classified as SP and SM according to the USCS. Type IV material is composed of reddish Havana silty clay with some moderately fine gravel and fine sand, classified as CL according to the USCS (**Figure 3**).

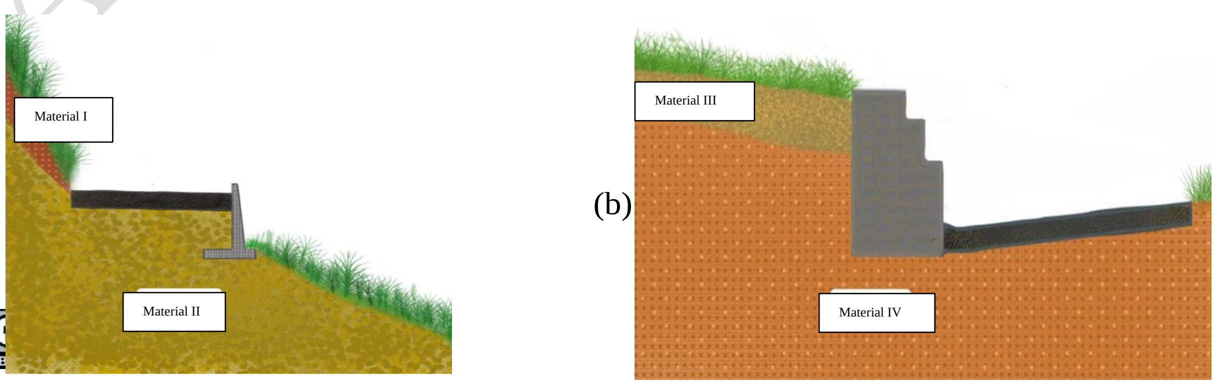

Figure 3 Stratigraphic soil profile of the rigid containment system (a) and flexible containment system (b). Source: Authors

So, **Table 5** displays the physical and mechanical parameters of the materials found in the two containment systems.

Table 5 Physical-mechanical soil parameters

Physical-mechanical parameter	Material I	Material II	Material III	Material IV
Total specific weight (kN/m3)	18.70	20.60	19.00	20.00
Friction angle (°)	30	22	30	23
Cohesion (kPa)	39.23	69.63	12.00	25.40
Specific gravity	2.65	2.40	2.65	2.75
Saturated specific weight (kN/m3)	21.80	22.07	21.37	20.54

Source: Authors

3.2 Factors Defining Disruptive Events

Due to Colombia's geographical location, geology, geomorphology, climate, among other factors, vary considerably depending on the region. For this case study, two main external factors triggering mass movements were considered: precipitation and seismicity.

3.3 Expected peak ground acceleration

The area where the two earth retaining systems are located is classified as having an intermediate seismic risk. For stability analysis purposes, a conservative value for maximum surface acceleration was adopted, based on Colombia’s seismic hazard study and current regulations, which establish a value of 0.50 for terrestrial materials. In this area, the estimated maximum effective acceleration is 0.20, considering a return period of 50 years..

3.4 Increases in water level associated with precipitation

Although both earth retaining systems are located on the same road, the sections where they are situated exhibit significant variations in their morphology and structural characteristics, due to the local climate and rainfall. According to state rainfall data, the rigid retaining system experiences an average annual precipitation of 1653 millimeters with an 80% relative humidity. The month with the highest precipitation is November, with a value of 211.2 millimeters. However, due to its geographical location, precipitation is significant in most months, with July being the driest month, receiving approximately 419.57 millimeters of rainfall. The flexible earth retaining system, on the other hand, has an average annual precipitation of 1117.45 millimeters and an 85% relative humidity. Precipitation in this region exhibits a bimodal pattern, with the rainiest periods occurring from March to May and from October to December, with a maximum annual precipitation of around 1850 millimeters. For this case study, the water table was conservatively estimated to be 50 cm below the ground surface for both containment systems. According to government sources, the water table under normal conditions varies between 2 meters and 5 meters from the surface.

4. Results

4.1 On-Site Inspection of the Rigid Earth Retaining System

From the on-site inspection process, it was determined that the system has a length of 8.85 meters and a height of approximately 4.83 meters, with a thickness of 0.31 meters (**Figure 4**) [7][35]. Additionally, the structure includes a Box Culvert at a depth of 1.20 meters, with dimensions of 1.20 meters in height and 0.60 meters in width, as well as three drainage works. Two of these drainage works are located at a depth of 2.50 meters, and the remaining one is situated at 4.60 meters, measured from the top of the containment wall. The drainage works are in good condition, except for one drainage that is completely covered with organic material. It was also observed that while water flows through all four hydraulic works, it also flows through various parts of the surface of the wall, leading to vegetation proliferation on the structure and accelerating its deterioration. Regarding the surroundings and vegetation, it is evident that vegetation predominates on the left side of the road, in the west-east direction, making it difficult to measure the slope of the embankment. Additionally, there is a perpendicular incline in relation to the terrain's surface. On the right side of the road, after the containment structure, the vegetation is completely vertical, indicating an absence of active land movements and the proper functioning of the structure. In summary, the road structure is generally in good condition, despite surface damage caused by an oval-shaped crack measuring 1.76 meters in length and 0.50 meters in width.

(a)

(b)

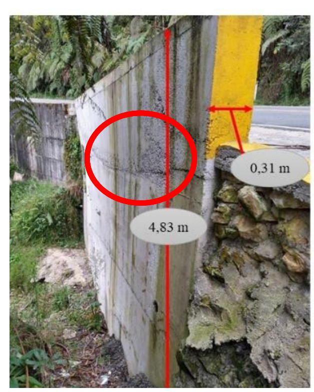

Figure 4 Characteristics of the rigid earth retention system in frontal view (a) and profile view (b). Source: Authors

In this context, after conducting a sclerometer test, the compressive strength of the retaining wall was determined, resulting in an average value of 29.50 MPa. Subsequently, when analyzing the pathologies of the retaining structure, it was found that aggregates and steel particles in the structure are visible. This indicates that the concrete was not adequately vibrated during the construction process. Infiltration of water was also observed, covering approximately 59.6% of the surface area of the retaining wall. The cracks and/or fissures on the surface of the wall cover approximately 8.34% of the total surface area of the structure.

4.2 On-Site Inspection of the Flexible Earth Retaining System

The flexible structure has a length of 38.8 meters and a height of 3.25 meters, which exhibited surface pathologies in the gabion mesh, primarily showing signs of oxidation (**Figure 5**). In the initial section of the first level of gabions in the east-west direction, the mesh has significantly yielded, causing the section to be non-linear and instead oblique. The mesh is also torn in some segments of the gabion, affecting approximately 5.20% of the total surface area of the structure. Additionally, about 50% of the structure is covered by vegetation, making it difficult to assess the condition of the mesh in this section of the structure and verify if it has any additional deformation. This containment system features two drainage structures, a gutter and an inspection box, both of which are in good condition and effectively fulfill their function.

(a) (b)

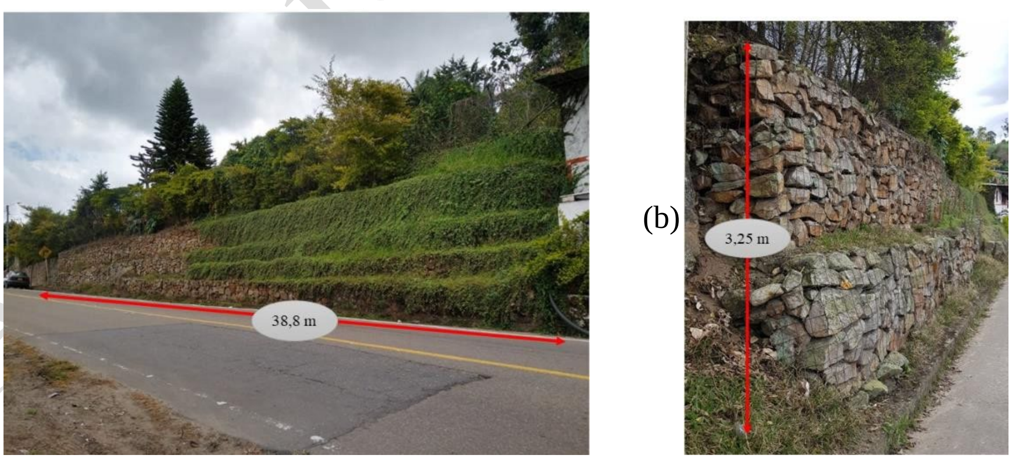

Figure 5 Characteristics of the flexible retaining system in a front view (a) and profile view (b). Source: Authors

Regarding the surroundings, the vegetation on the upper section of the structure is vertical. Since the upper area is part of an occupied lot, the vegetation in the area is periodically cleared and pruned. As for the slope, there is no evidence of active earth movements on the surface. Finally, the road pavement is well-preserved, although there is a slight separation between the road structure and the pavement, possibly due to a defective construction process. There is also evidence of maintenance and repairs to keep the road in good condition and functioning properly.

4.3 Resilience criteria test

Based on the characteristics and descriptions of each retaining system, and using Table 1 as a reference, each aspect included in the robustness criterion was assessed for the initial time (t_1) and the time following the occurrence of the disruptive event (t_2). **Table 6** presents the results of this evaluation for the two retaining systems analyzed.

Table 6 Evaluation of the robustness criterion in the rigid and flexible systems

Aspect	Rigid containment system		Flexible containment system	
	t_1	t_2	t_1	t_2
Rb_1: Structure inclination or displacement	0.70	0.40	0.70	0.00
Rb_2: Drainage works in the containment structure	0.40	0.00	0.70	0.00
Rb_3: Art works near the structure	0.70	0.70	1.00	1.00
Rb_4: Ground vegetation	0.70	0.40	1.00	0.40
Rb_5: State of the road	0.70	0.40	0.70	0.40
Rb_6: Concrete structure compression resistance	0.30	0.30	-	-
Rb_7: Concrete structure fissures and cracks	0.50	0.25	-	-
Rb_8: Concrete structure humidity	0.00	0.00	-	-
Rb_9: Flexible structure mesh condition	-	-	0.50	0.25

Source: Authors

In the redundancy criterion, for the reliability analysis under the four stability conditions, the software SLIDE6 used for data input relied on topographic studies (four cross-sections for each structure), the physical-mechanical properties of the subsurface materials, and the parameters defining disruptive events. Consequently, the model of analysis considered the angle of friction and cohesion of the materials in the subsurface as variables, resulting in the calculation of the safety factor and reliability index for each section. As an illustrative example, **Figure 6** depicts the first section of the structures for the most critical condition (high precipitation and seismic activity).

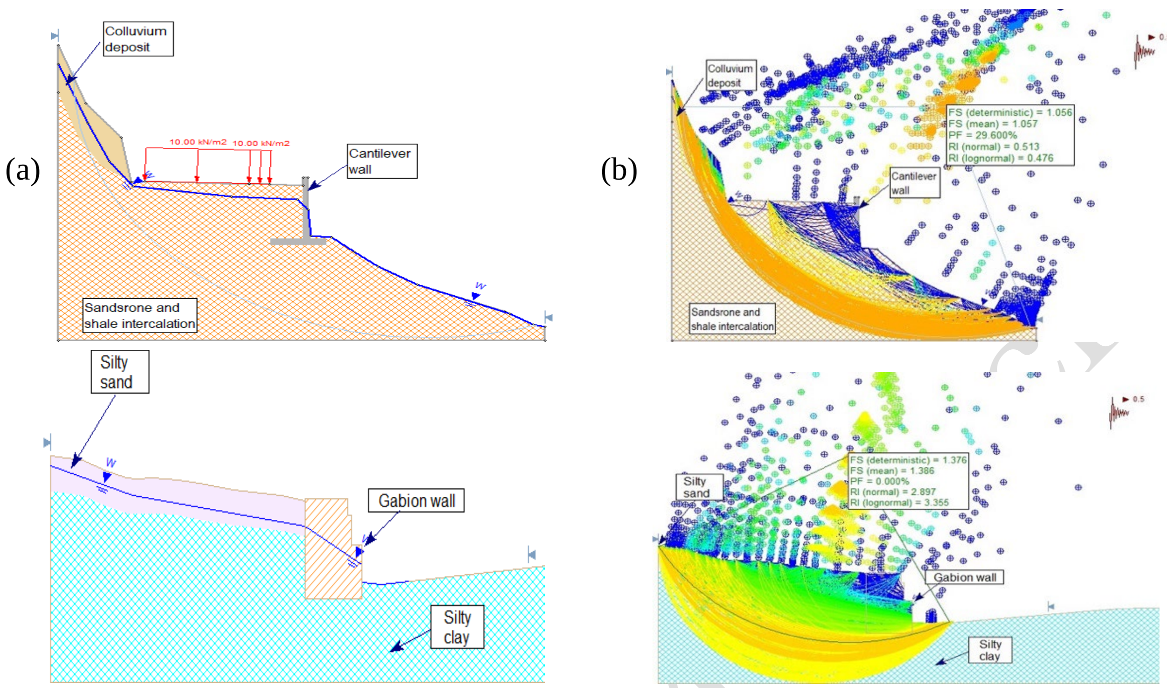

Figure 6 Result of the reliability analysis for redundancy estimation: modeling of the rigid system (a); modeling of the rigid system in critical condition (b); modeling of the flexible system (c); modeling of the flexible system in critical condition (d). Source: Authors

Based on the results of the reliability analysis, the factor of safety and reliability index of both systems were evaluated at times t_1 y t_2. For this purpose, the results of the normal condition were used to evaluate time t_1, and for t_2, the condition with the lowest factor of safety and reliability index was used, which in this case, for both systems, was the most critical condition with high precipitation and seismic activity.

Then, **Table 7** displays the assessment of redundancy aspects for both times analyzed, considering the evaluation presented in Table 2.

Table 7 Redundancy criterion (Rd) safety assessment results

Containment system	Safety index ($\boldsymbol{Rd_1}$)		Assessment	
	t_1	t_2	t_1	t_2
Rigid	1.891	0.912	0.70	0.00
Flexible	3.457	1.255	1.00	0.50

Source: Authors

To assess these aspects, it should be noted that both retaining systems are located along the same road corridor, which is managed by the same government agency. Therefore, the policies implemented for road and geotechnical work management are the same for both retaining systems, and they share most aspects of this criterion.

The economic activities in the municipalities connected by the road where both systems are located consist of livestock activities, including meat and dairy product trade, as well as agricultural activities, primarily strawberry and potato production. There are also some industrial activities, mainly in the construction sector.

Regarding the surrounding population affected by both systems, we directly and indirectly analyzed the total population of the municipalities connected by the road. The rigid retaining system is located near the municipality of Sibaté and connects several localities within the municipalities of Sibaté and Silvania. Based on this information and demographic reports from 2020, it is estimated that around 12,400 residents are affected. On the other hand, the flexible retaining system is located less than 2 kilometers from a relatively large municipality called Fusagasugá, and approximately 61,200 inhabitants were found to be affected.

Regarding the risk management budget, in terms of policies, it was observed that the road section where both systems are located is the responsibility of a Colombian government agency tasked with coordinating and allocating resources for risk identification, risk management, assessments, recovery activities in affected areas, and other tasks. This organization has digital tools for disaster risk management, enabling easy access to information through an interactive map on its website. Additionally, the organization allocates a budget for disaster risk management, expressed as a percentage of its total budget.

Three technical visits were conducted to assess the maintenance and condition of vegetation and drainage works, each lasting approximately two months. During these visits, it was confirmed that the vegetation was in an average state for the rigid retaining system since visual inspections indicated that cleaning and pruning work had been performed on the system. However, this maintenance was infrequent, and over time, the vegetation starts to cover the area near the structure, hindering proper surface water flow. For the flexible retaining system, it was evident that periodic cleaning and pruning are done every two months since the structure of this system is located under various occupied lots. Finally, it was observed that the drainage works were free from obstructions and clean during the inspections.

Lastly, taking into account the characteristics presented earlier for each aspect related to the ingenuity criterion, and based on the previously presented assessment, the evaluation of the components of this criterion was determined for the initial time (t_1) and the time after the occurrence of a disruptive event (t_2), and the results are summarized in **Table 8.**

Table 8 Resourcefulness (Rs) aspects assessment
Source: Authors

Aspect	Rigid containment system		Flexible Rigid containment system	
	t_1	t_2	t_1	t_2
Rs_1: Degree of the importance of the road according to economic activities	0.70	0.70	0.75	0.75
Rs_2: Risk management Budget	1.00	1.00	1.00	1.00
Rs_3: State of vegetation, in terms of maintenance	0.50	0.50	1.00	1.00
Rs_4: Drainage Works maintenance	0.33	0.33	1.00	1.00
Rs_5: Surrounding population	0.75	0.75	1.00	1.00
Rs_6: Type of road	0.67	0.67	0.67	0.67

To assess the aspects of rapid recovery (Re) presented earlier, a search was conducted for quarries relatively close to the studied systems. Subsequently, their minimum distance to the structures was determined, which, in the case of the rigid system, was 16.10 km, and for the flexible system, the minimum distance was 2.50 km. However, since the structures are located on the same road section, under the supervision of the same government organization, the evaluation of the social and economic impact aspects for both structures were the same.

For the social impact, the focus was on the impact on the population in terms of their activities. In this case, according to the Colombian National Institute of Highways (INVIAS), it is a secondary road, and the population's activities are primarily related to livestock trade and agricultural products. The government's municipal agencies are directly or indirectly affected by both retaining systems.

Finally, to assess the economic impact in terms of recovery, policies implemented by the road administration were analyzed through a quantitative analysis of resource allocation for the recovery of the structures. In this case, it was evident that the Special Unit for Risk Management of the Government of Cundinamarca (Colombia) has a budget allocated for emergency works and, in general, for risk management activities.

Based on the information presented above, each aspect of the recovery for each system was evaluated. However, it should be noted that the assessment of these aspects was the same at both times analyzed (t_1 and t_2), as these did not change in the event of a disruptive event. However, the final result of the recovery criterion does vary due to weighting factors that affect this criterion at the two times considered.

So, **Table 9** presents the results of the evaluation of each aspect for the two analyzed systems.

Table 9 Rapid recovery (Re) aspects assessment

Aspect	Rigid containment system		Flexible containment system	
	t_1	t_2	t_1	t_2
Re_1: Minimum distance to quarries for recovery	0.50	0.50	1.00	1.00
Re_2: Social impact in terms of recovery	0.75	0.75	0.75	0.75
Re_3: Economic impact in terms of recovery	1.00	1.00	1.00	1.00

Source: Authors

After evaluating each aspect of the resilience index criteria, the results for the two analyzed times were substituted into each criterion (equations 5-10). **Table 10** presents the consolidated results of the evaluation for each criterion, along with the adjustment factor as presented in equations (3) and (4), depending on the moment of interest.

Table 10 Evaluation of resilience index criteria.

Containment system	Time	Robustness (**Rb**)		Redundancy (**Rd**)		Resourcefulness (**Rs**)		Rapid recovery (**Re**)	
		Factor	Assessment	Factor	Assessment	Factor	Assessment	Factor	Assessment
Rigid	t_1	0.35	0.51	0.35	0.76	0.20	0.73	0.10	0.73
	t_2	0.15	0.31	0.20	0.00	0.30	0.73	0.35	0.73
Flexible	t_1	0.35	0.70	0.35	1.00	0.20	0.89	0.10	0.93
	t_2	0.15	0.26	0.20	0.38	0.30	0.89	0.35	0.93

Source: Authors

To evaluate the resilience index for both retaining systems, equations 2, 3, and 4 were used in conjunction with the results of the assessments of the robustness, redundancy, ingenuity, and rapid recovery criteria presented in Table 10. The outcome was the determination of the resilience value for each structure at times t_1 and t_2, and ultimately, the resilience index value for both systems (**Table 11**).

Table 11 Resilience index values for both earth retaining systems

Containment System	Time		Resilience Index (**RI**)
	t_1	t_2	
Rigid	0.662	0.517	0.782
Flexible	0.866	0.705	0.815

Source: Authors

5 Discussion

From the results of the case study, a comparison was made between the resilience index obtained from the analysis of the four criteria and the aspects contained within each of them. This comparison was not conducted directly, but it was presented within a general context, considering that both earth retaining systems are located in the same road section. The rationale behind this decision is that an earth retaining system is not directly comparable with the characteristics of the other because each system has been developed within a specific environment that contains intrinsic and extrinsic variables unique to its area.

This comparison aims to highlight the differences in the resilience index, despite being in the same road section and developed as part of the same project, which yielded different resilience values. In other words, the resilience index (RI) represents the recovery of the system with respect to its initial observed condition. As shown, the system with higher resilience capacity is the flexible earth retaining system, with an RI of 0.815, while the rigid earth retaining system has an RI of 0.782.

This result allows us to deduce that even if an earth retaining system is composed of a rigid concrete structure, it does not necessarily result in greater resilience for the system. On the other hand, resilience is a unique characteristic of each system. While there are similarities in the environment and type of structure, their resilience is different for each case because they interact with various economic, social, environmental, and technical aspects [6][8][9][27], which impact the degree of difficulty in recovery.

Based on the above, this study has demonstrated that the estimation and evaluation of resilience and its constituent criteria are inherent characteristics of the analyzed earth retaining systems. Therefore, to assess the resilience of a system, it is necessary to analyze the various variables involved that exert both direct and indirect influences on the entire system.

Regarding the proposed equations for evaluating resilience in geotechnical engineering, especially the equations presented, it is essential to highlight that this initial approach allows for the determination and evaluation of the resilience criteria, as well as obtaining a resilience value with simplicity while maintaining coherence in the analysis.

This is because the resilience components of a system are systematized based on tables with fixed values, determined according to technical inspections of the structures and the environment, which can be carried out at any time and do not require specialized equipment and/or highly qualified personnel. This accessibility ensures that the analysis can be easily applied to any geotechnical project or by entities interested in assessing the resilience of geotechnical containment systems, enabling them to make informed decisions about implementing preventive and corrective measures or formulating policies to enhance and maintain resilient systems.

6 Conclusions

There is no doubt that the concept of resilience has become the focus of multidisciplinary studies in recent years, aiming to reduce intrasystemic entropy (related to the intrinsic properties of the system of interest), intersystemic entropy (resulting from its interaction with other systems), and extrasystemic entropy (caused by external forces that destabilize its original condition). For this reason, a theoretical and conceptual understanding of resilience, as well as its scope and limitations, is valuable for researchers' studying systems across various disciplines.

In this regard, the ease of obtaining a resilience index is particularly useful, given that these types of systems are frequently used in developing countries, where regulatory bodies responsible for geotechnical infrastructure oversight are often lacking. Furthermore, governmental agencies in these regions typically do not have sufficient resources to implement measures that reduce the entropy affecting such systems. Therefore, resilience analysis can support decision-making in the sector. In fact, this case study examined two relatively small but widely used earth retention systems in Latin America. Assessing the resilience index allows for the design, construction, and optimization of systems that remain sustainable over time. This is justified by the fact that such an assessment is not solely based on the physical, mechanical, and geotechnical characteristics of systemic structures but also on a set of variables that directly and indirectly influence their functionality and are critical in the face of disruptive events.

Finally, as a prospective aspect of this research, it can contribute to broader analyses that encompass the behavior of different retention systems across various study areas. For example, in the case of Colombia, the goal was to refine and propose a more precise approach to the equations governing the resilience component. Therefore, the criteria, resilience index, and the considerations made for each criterion—based on expert judgment—become highly relevant.

In this context, the evaluation results can help identify areas for improving geotechnical infrastructure and addressing extreme events, even in the face of climate change, which introduces greater uncertainty into road project planning. Additionally, as mentioned earlier, this research can support more in-depth analyses that facilitate the systematization of calculations using various software tools, enabling a comprehensive and rapid determination of the resilience index for any containment system based on a more robust database.

Declaration of competing interest

We declare that we have no significant competing interests including financial or non-financial, professional, or personal interests interfering with the full and objective presentation of the work described in this manuscript.

Acknowledgement

The authors would like to express their deep gratitude to the panel participating experts for their time and contributions to this research.

Funding

The authors received no financial support for the research, authorship, and/or publication of this article.

Author contributions

J. A. P. Formal analysis, Investigation, Methodology, Writing. L. S. Investigation, Resources, Writing. M. G. G. P. Conceptualization, Writing

References

[1] ASCE, 2000., 2003. Prestandard and Commentary for the Seismic Rehabilitation of Building. FEMA 356. [WWW Document]. https://www.nehrp.gov/pdf/fema356.pdf

[2] Basu, D., and Mina L., 2018. A Quantitative Framework for Sustainability and Resilience in Geotechnical Engineering. In A. Murali Krishna, A. Dey, and S. Sreedeep (Eds), Geotechnics for Natural and Engineered Sustainable Technologies. https://doi.org/10.1007/978-981-10-7721-0_24.

[3] Tighnavard, B, A. and Marsono. A., 2020. Applying Multi-Criteria Decision-Making on Alternatives for Earth-Retaining Walls: LCA, LCC, and S-LCA. International Journal of Life Cycle Assessment 25 (11), 2140-2153. https://doi.org/10.1007/s11367-020-01825-6

[4] Ali R., Amir H. Gandomi, Koorosh Azizi, and Charles V. Camp. 2022. Multi-Objective Optimization of Reinforced Concrete Cantilever Retaining Wall: A Comparative Study." Structural and Multidisciplinary Optimization 65 (9). https://doi.org/10.1007/s00158-022-03318-6.

[5] Bocchini, P. and Frangopol, D. M., 2011. Resilience-Driven Disaster Management of Civil Infrastructure. In M. Papadrakakis, M. Fragiadakis, V. Plevris (Eds.), Conference: Computational Methods in Structural Dynamics and Earthquake Engineering. [WWW Document]. https://congress.cimne.com/eccomas/proceedings/ compdyn2011/compdyn2011_full/473.pdf

[6] Basu, D., Aditi M., and Anand J. P., 2014. Sustainability and Geotechnical Engineering: Perspectives and Review. Canadian Geotechnical Journal 52 (1), 96-113. https://doi.org/10.1139/cgj-2013-0120

[7] Bonilla, Manuela, and Rincon Andrea., 2021. Resilience assessment in the road Sibate-San Miguel, Colombia. BSc Civil Engineering conclusion work. Universidad Distrital Francisco Jose de Caldas, Bogota. 145 pp. In Spanish.

[8] Pineda, J. A. 2019. "Weighting Factors for Estimating Resilience Index in Earth Retaining Systems in Colombia." Postodoctoral Fellow at Research Center for Sustainable Engineering (CIDIS) Working Paper.

[9] Broniewicz, E. and Ogrodnik, K., 2020. Multi-Criteria Analysis of Transport Infrastructure Projects. Transportation Research Part D: Transport and Environment 83, 1-15. https://doi.org/10.1016/j.trd.2020.102351.

[10] Broniewicz, E. and Ogrodnik, 2021. A Comparative Evaluation of Multi-Criteria Analysis Methods for Sustainable Transport. Energies 14 (16), 5100. https://doi.org/10.3390/en14165100.

[11] Bruneau M., Chang S. E., Eguchi, R. T., Lee, G. C., O'Rourke T. D., Reinhorn, A. M., Shinozuka, M., Tierney, K. & Wallace, W. A. and Winterfeldt, D. V., 2003. A Framework to Quantitatively Assess and Enhance the Seismic Resilience of Communities. Earthquake Spectra 19 (4): 733–52. https://doi.org/10.1193/1.1623497.

[12] Holling, C S., 1973. Resilience and Stability of Ecological Systems. Annual Review of Ecology and Systematics 4 (1), 1–23. https://doi.org/10.1146/ANNUREV.ES.04.110173.000245.

[13] Calvente, A., 2007. Resiliencia: un concepto clave para la sustentabilidad. UAIS Sustentabilidad, 1-4. [WWW Document]. http://sustentabilidad.uai.edu.ar/pdf/cs/UAIS-CS-200-003%20-%20Resiliencia.pdf

[14] Pimm, S. L., 1984. The Complexity and Stability of Ecosystems. Nature 307 (5949). https://doi.org/10.1038/307321a0.

[15] Howell, J. R., and Buckius, R. O., 1990. Principios de termodinámica para ingenieros. McGraw-Hill

[16] González, M. G., 2018. Entropy and negentropy of private electric vehicles in urban systems: homeostasis of mobility in Mexico. DYNA, 85(206), pp. 171-177. https://doi.org/10.15446/dyna.v85n206.72509

[17] Carnot, S., 1872. Reflexions sur la puissance motrice du feu et sur les machines propres dvelopper cette puissance. Annales scientifiques de l'.cole Normale Sup.rieure. Serie 2, Tome 1, 393-457. [WWW Document]. http://www.numdam.org/article/ASENS_1872 _2_1__393_0.pdf

[18] Varadhan, R., 2015. Entropy and many avatars. Journal Mathematic Social Japan 67(4), 1845-1857. https://doi.org/10.2969/jmsj/06741845

[19] Schrödinger, E., 1944. What is life?. University Press.

[20] Shannon, C. and W. Weaver, W., 1949. The mathematical theory of communication. University of Illinois Press.

[21] Bertalanffy, L. V.,1968. General system theory. Foundations, Development, Applications. George Braziller.

[22] Kish, L. and Ferry D., 2017. Information entropy and thermal entropy: apples and oranges. Journal of Computational Electronics 1(8), 1- 8. [WWW Document]. https://arxiv.org/pdf/1706. 01459.pdf

[23] Carpenter S., Walker B., Anderies J. M., and Nick N., 2001. From Metaphor to Measurement: Resilience of What to What?. Ecosystems, 4, 765-781. https://doi.org/10.1007/s10021-001-0045-9

[24] Rosowsky, David V. 2020. Defining Resilience. Sustainable and Resilient Infrastructure 5 (3).

https://doi.org/10.1080/23789689.2019.1578166.

[25] Simpson, B., 1992. Retaining Structures: Displacement and Design. Geotechnique 42 (4) 541-576. https://doi.org/10.1680/geot.1992.42.4.541.

[26] Salgado, Rodrigo, 2022. The Engineering of Foundations, Slopes and Retaining Structures. CRC Press.

[27] Cimellaro, G. P., 2013. Resilience-based design (RBD) modelling of civil infrastructure to assess seismic hazards, Editor(s): S. Tesfamariam, K. Goda, In Woodhead Publishing Series in Civil and Structural Engineering, Handbook of Seismic Risk Analysis and Management of Civil Infrastructure Systems, Woodhead Publishing. https://doi.org/10.1533/9780857098986.2.268.

[28] Kermanshachi, S., Rouhanizadeh, B. and Dao, B., 2019. Application of Delphi Method in Identification, Ranking, and Weighting of Project Complexity Indicators for Construction Projects. Journal of Legal Affairs and Dispute Resolution in Engineering and Construction 12(1). https://doi.org/10.1061/(ASCE)LA.1943-4170.0000338

[29] Trujillo, R., 2004. Aplicaciones del método Delphi: Casos exitosos de forecasing en Colombia (Empaques flexibles y semi rígidos en Colombia y Cadena láctea y sus derivados en Colombia. Universidad Externado de Colombia.

[30] Oñates N. and Martínez L., 1990. Utilización del método Delphy en la pronosticación: Una experiencia inicial. Instituto de Investigaciones Económicas.

[31] Saint Paul, R. and Ténière, P. F. (1974). Innovation et évaluation technologiques. Entreprise moderne d'édition. García V., Aquino S., P., Guzmán A. and Medina, A., 2011. Propuesta para el desarrollo de instrumentos de autoevaluación para programas educativos a distancia. Revista Electrónica: Actualidades Investigativas en Educación, 11(2), 1-27. [WWW Document]. https://www.redalyc.org/pdf/447/44720020017.pdf

[32] Astigarraga, E., Método Delphi. Universidad de DeustFrancis, R. and Behailu B., 2014. A Metric and Frameworks for Resilience Analysis of Engineered and Infrastructure Systems. Reliability Engineering and System Safety 121, 90-103. https://doi.org/10.1016/j.ress.2013.07.004

[33] Pineda, J.A.; Larrañaga, S.C.; Carmona, J.E., 2024. Multi-criteria Analysis for the Estimation of Resilience Indices in Geotechnical Systems: Application to Earth-Retaining Structures. The International Journal of Architectonic, Spatial, and Environmental Design. 18(2). https://doi.org/10.18848/2325-1662/CGP/v18i02/43-71

[34] Pineda-Jaimes, J.A., García-Ubaque, C.A. and Esquivel-Ramírez, R.E., 2020. Assessment of Geotechnical Hazard due to Deep Excavations in Bogota Clays: A Contribution for Sustainability in Urban Environments, Rev. Fac. Ing., 29(54), p. e11373. doi.org/10.19053/01211129.v29.n54.2020.11373

[35] Delgado, Yuli A., Jorge A. Pineda, and Gabriel J. Colorado-Urrea., 2024. Resilience IndexAssessment for Urban Excavations. In Procedings of the Geocongress American Society of Civil Engineers, 709–720. https://doi.org/10.1061/9780784485316.072