Annexes to "Economic Growth Consequences of Structural Stagnation: A Two-Sector Model of Productive Diversification" by Carlos H. Ortiz

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Annex 1: The Elasticity of Substitution between Intermediate Inputs

From equation (2), it is deduced that the marginal productivity of the input X_{ij} on the output of the *j*-th good is given by

$$\frac{\partial X_j}{\partial X_{ij}} = (1-\alpha) K_j^{\alpha} X_{ij}^{-\alpha} .$$

Cost minimization of good X_j implies that the ratio of marginal productivities of goods X_{ij} and X_{jj} should be equal to the respective price ratio

$$X_{ij}^{-\alpha} \big/ X_{jj}^{-\alpha} = p_i \big/ p_j \ .$$

Thus, the elasticity of technical substitution between goods *i* and *j* is given by

$$-\frac{\partial (X_{ij}/X_{jj})}{\partial (p_i/p_j)} \frac{(p_i/p_j)}{(X_{ij}/X_{jj})} = \frac{1}{\alpha} > 1 .$$

Annex 2: Small Country Dynamics under Capital Price Equalization

The Hamiltonian related to the dynamic intertemporal problem of the specific country is given by

$$H = e^{-\rho t} \log(D) + m(r_K K + R - p^* D) .$$

The first order conditions (FOCs) are

$$H_D = 0 \ \therefore \ e^{-\rho t} D^{-1} = m p^* \ \therefore \ -\rho - g_D = g_m + g_{p^*}$$

$$\dot{m} = -H_K \ \therefore \ \dot{m} = -m r_K \ \therefore \ g_m = -r_K$$

$$\dot{K} = H_m \ \therefore \ \dot{K} = r_K K + R - p^* D$$

$$\lim_{t \to \infty} mK = 0$$

From the previous FOCs, it is deduced that the value of food demand, p^*D, grows at the same rate as capital accumulation at the world level:

$$g_{P^*} + g_{D^*} = r_K - \rho = AN^* - \rho - \delta = g_{K^*} .$$

Thus, the value of food demand evolves according to the following time path:

$$p^* D = p_0^* D_0 \exp[AN_0^* (e^{\nu t} - 1)/\nu - (\rho + \delta)] ,$$

where $p_0^* D_0$ is the initial value of food demand.

The capital path is deduced by integrating the corresponding differential equation (the third FOC). As some variables can be expressed in terms of their fundamentals, this differential equation may be rewritten as follows:

$$\dot{K} - (AN^* - \delta)K = \theta K^* - p^* D .$$

The corresponding integration factor is given by

$$f = \exp\left[- \int_0^t (AN_0^* e^{\nu s} - \delta)\, ds \right] = \exp\left[- AN_0^* (e^{\nu t} - 1)/\nu + \delta t \right].$$

Multiplying the differential equation throughout by f, writing the left-hand-side expression of this equation in a time derivative form [= $d(r_K K)/ds$], and integrating afterwards between 0 and t yields

$$K \exp[-AN_0^* (e^{\nu t} - 1)/\nu + \delta t] = K_0 + (\theta K_0^* - p_0^* D_0)(e^{-\rho t} - 1)/(-\rho).$$

Given the transversality condition (the fourth FOC), the left-hand side expression of this equation must vanish in the infinity. In order to guarantee that outcome, the initial value of food demand must be determined as follows:

$$p_0^* D_0 = \rho K_0 + \theta K_0^* .$$

Substituting this result in the previous expression yields the mathematical expression for the capital path:

$$K = K_0 \exp[AN_0^* (e^{\nu t} - 1)/\nu - (\delta + \rho)t] .$$

Annex 3: A Hypothetical World Equilibrium with International Price Differentials

In order to have an insight into this phenomenon, let us consider the process of structural transformation in a dualistic world: many nations in the South are underdeveloped with the same degree of manufacturing diversification (N_0), and a few nations in the North are developed with the same degree of manufacturing diversification (N_0^*).

Figure A1

World Structural Change

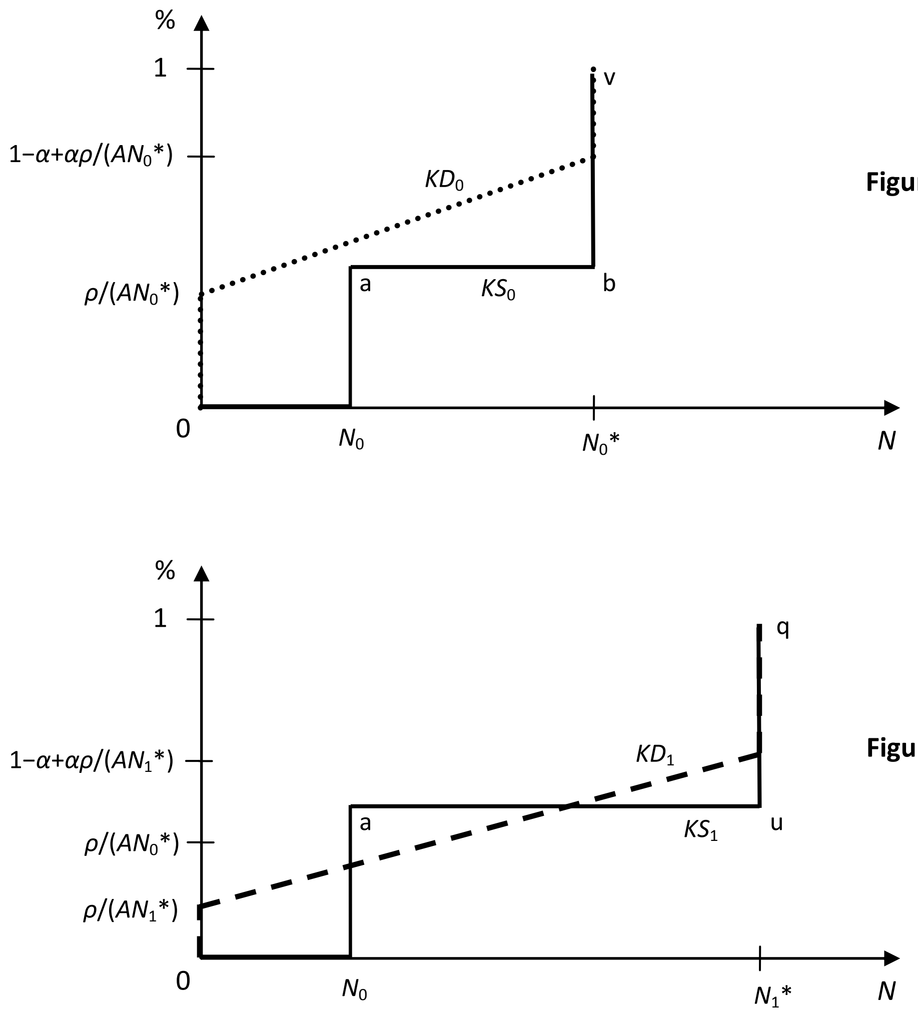

Figure A1.0

Figure A1.1

Source: the author.

Figure A1 depicts the world structural change from period 0 to period 1. Figure A1 shows that the initial distribution of cumulated capital supply is given by the continuous line KS_0 (line 0-$\boldsymbol{N}_0$-a-b-v): the relative capital endowment of the South is given by the continuous line at the level of $\boldsymbol{N}_0$ (line $\boldsymbol{N}_0$-a), and the remainder –the relative capital endowment of the North– is given by the continuous line at the level of N_0^* (line b-v). On the other hand, the initial distribution of cumulated capital demand is given by the dotted line KD_0. It corresponds to the distribution of capital demand that was explained in Figure 5 in the paper. Because of its low degree of manufacturing diversification, the South only produces goods characterized by indices of technological diversification below or equal to $\boldsymbol{N}_0$. In this initial situation, there is not complete specialization because the North has to produce some of these goods: the South´s supply of capital is lower than the required international demand for capital for the production of goods indexed up to N_0. Thus, some goods are commonly produced so that price equalization holds for goods and capital across countries. In this situation, as it was previously proved, the North and the South grow at the same pace because the South is functionally articulated to the international economic system: although the growth engine of the world economy is the diversification process that the North monopolizes, the South shares the benefits through international trade.

As it was shown before, under factor price equalization no underdeveloped country experiences the need of keeping itself along the track of productive diversification. Hence, let us assume that in a further stage –period 1– the Southern degree of technological diversification remains at $\boldsymbol{N}_0$, whilst the Northern degree increases from N_0^* to N_1^*. Hence, as Figure A1.1 depicts, the cumulated capital demand distribution lowers (from KD_0) to KD_1, the dashed line. This movement represents the world structural change from period 0 to period 1. On the other hand, the cumulated capital supply distribution changes to KS_1 (line 0-$\boldsymbol{N}_0$-a-u-q). Notice, however, that the South´s relative supply of capital is unchanged (it is given by the same line $\boldsymbol{N}_0$-a). The explanation is the following: in the initial situation the factor price equalization theorem holds, and, as previously proved, capital grows everywhere at the same pace. Now, in period 1 the factor price equalization theorem does not hold any longer, since the world technological gap is now wider ($N_1^* > N_0^* > N_0$), a lower relative level of capital is demanded for primary activities in the world economy [$\rho/(AN_1^*) < \rho/(AN_0^*)$]. Therefore, a sufficiently wide gap in technological diversification between North and South may cause disequilibrium in the international capital market. As Figure A1.1 shows, in period 1 the Southern capital supply is higher than the international capital demand for productive activities whose degree of technological integration is less than or equal to N_0. What is happening seems to be a paradox: the underdeveloped South has an excessive supply of capital… given its low degree of technological diversification. The Southern excess supply of capital is depicted by the gap between the continuous line KS_1 at the level N_0, and the dashed line KD_1 at the same level. This excess of capital would like to flee to the North, and some of it may be successful in this endeavour –as the migration flows from underdeveloped to developed countries reveal, but the huge barriers to factor international mobility avoid the total suppression of the disequilibrium. In this situation, the South is completely specialized in activities of lower technological integration, and the North is completely specialized in activities of higher technological integration. Initially, there may be unemployed capital in the South, but sooner or later the excess supply of capital lowers the Southern capital gross remuneration, and with it also lowers the Southern goods prices. Recall, as equation (9) shows, that there exists a direct relationship between capital gross remuneration and relative prices.

Annex 4: Small Country Dynamics under Unequal Capital Price

The intertemporal objective function is as before

$$U_0 = \int_0^{\infty} e^{-\rho t} \log(D)\, dt \, .$$

This objective function is maximized subject to the instantaneous budget constraint

$$p^* D + \dot{K} + \delta K = (r_{K^*} + \delta) K + R \, ,$$

where $r_{K^*} + \delta = \gamma A N^*$ and $R = \psi K^*$. The budget constraint holds at every moment in time.

The related Hamiltonian equation is

$$H = e^{-\rho t} \log(D) + m(r_K K + R - p^* D) \, .$$

The first order conditions are

$$H_D = 0 \;\; \therefore \;\; e^{-\rho t} D^{-1} = m\, p^*$$

$$\dot{m} = -H_K \;\; \therefore \;\; \dot{m} = -m r_K$$

$$\dot{K} = H_m \;\; \therefore \;\; \dot{K} = r_K K + R - p^* D$$

$$\lim_{t \to \infty} mK = 0$$

As it was done before, from these FOCs it is deduced that the value of food demand, p^*D, grows at the following pace:

$$g_{p^*} + g_D = r_K - \rho = \gamma A N^* - \rho - \delta.$$

Hence, in this situation the specific country´s value of food demand expands at a lower rate than capital of the representative world country ($g_{p^*} + g_D = \gamma A N^* - \rho - \delta < g_{K^*} = A N^* - \rho - \delta$).

From the third FOC, it is deduced that

$$m = m_0 \exp(-r_K t) = m_0 \exp[-(\gamma A N^* - \delta)t].$$

For this result, it is convenient to recall that the specific country remuneration factor is assumed to be constant: r_K $(= \gamma A N^* - \delta)$ is constant as N^* is assumed to be constant.

The capital differential equation is given by

$$\dot{K} - (\gamma AN^* - \delta)K = \psi K^* + p^* D\ .$$

This is a first-order linear differential equation whose integration factor is given by

$$f = \exp[(-\gamma AN^* + \delta)t]\ .$$

Multiplying the differential equation throughout by *f*, writing the left-hand-side expression in a time derivative form [= $d(r_K K)/ds$], and integrating afterwards between 0 and *t* yields

$$K \exp\left[-\left(\gamma AN^* - \delta\right)t\right] = K_0 + \psi K_0^* \frac{\exp\left[\left((1-\gamma)AN^* - \rho\right)t\right] - 1}{(1-\gamma)AN^* - \rho} + p_0^* D_0 \left[\exp(-\rho t) - 1\right]/\rho .$$

For the transversality condition to hold (the fourth FOC), the left-hand-side expression of the equation above must vanish in the infinity. This condition is satisfied if and only if

$$(1-\gamma)AN^* - \rho\ <\ 0\ .$$

Hence, the following chain of inequalities must hold: $0 < 1 - \rho/(AN^*) \equiv 1 - z^* < \gamma < 1$. The terms of trade deterioration index, γ, cannot fall behind the capital allocation fraction to manufacturing activities in the world, $1 - z^*$. On the other hand, the transversality condition requires that the initial value of food demand satisfies the following equation:

$$P_0^* D_0 = \rho K_0 - \frac{\rho \psi K_0^*}{(1-\gamma)AN^* - \rho}\ .$$

The capital time path is then defined by

$$K = K_0 \exp[(\gamma AN^* - \delta - \rho)t] - \frac{\psi K_0^*}{(1-\gamma)AN^* - \rho}\left\{\exp[(\gamma AN^* - \delta - \rho)t] - \exp[(AN^* - \delta - \rho)t]\right\},$$

so that capital accumulation grows at the following rate:

$$g_K = \left[\frac{K_0 \exp(g\,t) - \Omega[(g_{K*}/g)\exp(g_{K*}t) - \exp(g\,t)]}{K_0 \exp(g\,t) - \Omega[\exp(g_{K*}t - \exp(g\,t)]}\right] g\ ,$$

where $\Omega \equiv [-\psi K_0^*]/[(1-\gamma)AN^* - \rho] > 0$, $g \equiv \gamma AN^* - \delta - \rho$, and $g_{K*} \equiv AN^* - \delta - \rho$.