

# An Optimal Choice among Pension Systems: Evidence from Colombia

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**Abstract:** This paper examines how Colombian workers choose between two pension regimes, pay-as-you-go (PAYG) and individual account (IA), in a context marked by widespread labor informality and uneven wage distributions. Drawing on a discrete choice framework, we hypothesize that there exists a critical wage threshold above which the higher replacement rate offered by PAYG outweighs the lower accessibility constraints. In contrast, while below this threshold, the IA system's more generous lump sum payouts become more appealing for individuals who may struggle to meet minimum contribution requirements. The model is calibrated to the data and then subjected to Monte Carlo simulations to gauge consistency with observed system preferences. Findings indicate that PAYG tends to favor higher earners through regressive subsidies and stricter qualification criteria, whereas IA proves more accommodating to low-income earners. The results underscore the need for nuanced policy reforms that acknowledge these distributional imbalances and address the challenges of labor informality.

**Keywords:** Colombian pension system, individual account pension systems, optimal choice, Pay as You Go pension system, regressivity.

**JEL Classification:** H55, J26, J32.

## Una elección óptima entre sistemas de pensiones: Evidencia de Colombia

**Resumen:** Este artículo explora cómo los trabajadores colombianos eligen entre dos regímenes de pensiones, el de prima media (RPM) y el de ahorro individual (RAIS), en un contexto marcado por una amplia informalidad laboral y distribuciones salariales desiguales. Basados en un marco de elección discreta, planteamos la hipótesis de que existe un umbral salarial crítico por encima del cual la mayor tasa de reemplazo ofrecida por el RPM supera las menores restricciones de accesibilidad, mientras que, por debajo de este umbral, los pagos únicos más generosos del RAIS se vuelven más atractivos para quienes pueden tener dificultades para cumplir con los requisitos mínimos de cotización. El modelo se calibra con datos y luego se somete a simulaciones de Monte Carlo para evaluar su consistencia con las preferencias observadas en el sistema. Los hallazgos indican que el RPM tiende a favorecer a los afiliados de mayores ingresos a través de subsidios regresivos y criterios de calificación más estrictos, mientras que el RAIS se adapta mejor a los de

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bajos ingresos. Los resultados subrayan la necesidad de reformas pensionales matizadas que reconozcan estos desequilibrios distributivos y aborden los desafíos de la informalidad laboral.

**Palabras clave:** Sistema pensional colombiano, Sistemas de pensiones de capitalización individual, Elección óptima, Sistema de pensiones de reparto, Regresividad.

## **Un choix optimal parmi les systèmes de retraite : Le cas de la Colombie**

**Résumé:** Cet article explore comment les travailleurs colombiens choisissent entre deux régimes de retraite, le régime par répartition (RPM) et le régime de capitalisation individuelle (RAIS), dans un contexte marqué par une informalité généralisée du travail et des distributions de salaires inégales. En nous appuyant sur un cadre de choix discret, nous posons l'hypothèse qu'il existe un seuil de salaire critique au-dessus duquel le taux de remplacement plus élevé offert par le régime par répartition l'emporte sur les contraintes d'accessibilité moindres, tandis qu'en dessous de ce seuil, les versements forfaitaires plus généreux du système de capitalisation individuelle deviennent plus attrayants pour les personnes qui pourraient avoir du mal à satisfaire aux exigences minimales de cotisation. Le modèle est calibré à partir de données, puis soumis à des simulations de Monte Carlo pour évaluer sa cohérence avec les préférences observées dans le système. Les résultats indiquent que le régime par répartition a tendance à favoriser les salariés à revenus élevés par le biais de subventions régressives et de critères de qualification plus stricts, alors que le régime de capitalisation individuelle s'avère plus accommodant pour les personnes à faibles revenus. Les résultats soulignent la nécessité de réformes politiques nuancées qui reconnaissent ces déséquilibres distributifs et relèvent les défis de l'informalité du travail.

**Mots-clés :** Système de retraite colombien, Systèmes de retraite à capitalisation individuelle, Choix optimal, Système de retraite par répartition, Régressivité.

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**-Introduction. -I. Theoretical Framework. -II. Model. -III. Monte Carlo Experiment. - Conclusions. -Acknowledgements. -Ethics Statement. -Appendix. -References.**

## Introduction

Colombia's pension system has undergone significant transformations over the past several decades, evolving in tandem with the country's changing socioeconomic conditions. Despite such reforms, two persistent and interrelated challenges remain: ensuring fair coverage across different income levels and coping with the reality of widespread labor informality. In particular, Colombia offers two formal pension regimes: pay-as-you-go (PAYG) and individually funded (IA). Workers must choose which system to join under conditions that involve varying degrees of income stability, contribution density, and expected replacement rates.

This paper addresses an important research gap: how do individuals optimally choose between PAYG and IA in an environment characterized by pronounced labor informality and uneven wage distributions? We hypothesize that there exists a critical wage threshold that effectively delineates which pension system is more advantageous. Above this threshold, the higher replacement rate of the PAYG system tends to dominate, whereas below it, the IA system's more generous lump sum reimbursement for those who fail to meet minimum contribution requirements becomes more appealing.

Building on this hypothesis, we pose two central questions:

1. How do high replacement rates and labor informality, reflected in low contribution densities, shape an individual's selection of a pension system?
2. How does an individual's choice vary with income levels when confronted with these two regimes?

To answer these questions, first, we develop a discrete choice model calibrated to Colombian data to rigorously identify how income levels, contribution density, and replacement rates interplay in driving pension system selection. And then, we introduce a Monte Carlo simulation approach that replicates observed patterns in the Colombian pension environment, most notably, the percentage of individuals who opt for the IA system and the average number of contributions made at retirement, while also revealing the regressivity inherent in PAYG subsidies. In doing so, we not only replicate key empirical features of Colombia's pension landscape (such as the proportion of individuals opting for IA and the number of contributions made before retirement) but also demonstrate how the intrinsic design of each system interacts with labor informality to generate distinctly different distributional effects.

This contribution is significant from multiple standpoints. Academically, it bridges a gap in literature by combining the realities of informal labor markets with the theoretical frameworks of pension choice under uncertainty. From a policy perspective, the findings elucidate how varying wage levels and contribution patterns can exacerbate or mitigate regressive outcomes in the PAYG system. Understanding these dynamics can aid in designing reforms aimed at sustainability and equity, especially where large segments of the workforce are informally employed.

This paper is designed in the following ways. In the first part, we describe the Colombian context along with the literature review in Section I. In this section, a historical context and theoretical foundations, along with a review of the Literature from various author contributions and shortcomings of the past studies, have been provided. The methodology adopted and the findings of the proposed model are discussed in the next section. In Section III, we carried out model

calibration and simulations. Final Section IV outlines the key conclusions, limitations, and suggests a direction for further research.

### ***A. Description of Colombian context***

The pension system in Colombia has evolved since the establishment of entities such as CAJANAL and ICSS in the 1940s, founded on principles of collective solidarity and mandatory employer responsibility. In 1967, the ICSS adopted the Average Premium with Defined Benefit Regime (Régimen de Prima Media con Prestación Definida, RPM), a pay-as-you-go scheme where pensions are financed by contributions from active workers. This model prevailed until the enactment of Law 100 of 1993, which introduced the General Pension System (Sistema General de Pensiones, SGP) with two regimes: the RPM (later managed by Colpensiones) and the Individual Savings with Solidarity Regime (Régimen de Ahorro Individual con Solidaridad, RAIS), along with the creation of the Minimum Pension Guarantee Fund (Fondo de Garantía de Pensión Mínima, FGPM) to support low-income affiliates in the RAIS.

In both regimes, financing comes from contributions and transfers from the National Government, although the allocation of these resources varies by regime. For dependent employees, contributions amount to 16% of salary (12% employer, 4% employee); independent workers under service contracts base their pension contributions on 40% of their monthly invoiced income (excluding VAT) (Government of Colombia, Decree 1273 of 2018, art. 3.2.7.1), paying the full 16% via the Integrated Contribution Form (Planilla Integrada de Liquidación de Aportes, PILA). The RPM manages a collective fund and applies a formula based on average salary and weeks of contributions. In RAIS, contributions are deposited into individual accounts, and pensions depend on capital accumulation and their returns, with a state subsidy for low-income workers.

Pension requirements in the RPM include a minimum age (62 for men, 57 for women) and 1,300 weeks of contributions. Those who fail to meet these requirements may receive a substitute of indemnity (return of contributions adjusted for inflation). In the RAIS, 1,150 weeks are required, along with sufficient capital to fund at least 110% of the legal minimum wage. Flexible pension options are available (e.g., programmed withdrawal, life annuity) or a balance refund.

While RPM offers higher replacement rates, its equity is questionable. High levels of informality and difficulty in meeting the minimum weeks of contributions lead to the exclusion of low-income workers who are left without a pension and receive smaller refunds. The RPM's financial deficit is increasing, relying on the national budget (42.7 trillion pesos in 2019), raising concerns about its sustainability (Compite Caribe, 2020).

RAIS, with a lower replacement rate, provides greater sustainability by relying on individual savings. However, it faces market risks and irregular contributions due to labor informality. As a result, the current dilemma involves the need for reforms to balance equity, coverage, and financial viability in the context of high informality and an aging population.

## **I. Theoretical Framework**

Research in the field of optimal pension system choice is a complex process because it needs a deeper understanding of the economic, behavioral, and societal factors that affect individual decisions. In this section, an attempt has been made to outline important contemporary research to

develop arguments regarding the choices made by individuals in their pension system selection process. While the authors have tried to be more specific to the Colombian context, an overall framework to understand mixed Individual Account (IA) and Pay-As-You-Go (PAYG) systems has been presented. Investigations have expanded the life-cycle hypothesis framework by demonstrating how different saving mechanisms alter consumption smoothing in pension decisions, revealing variation across demographic groups (Angerer et al., 2025).

The first important concept underlying this framework is the Life-Cycle Hypothesis, which was first given by Modigliani and Brumberg (1954). Ando and Modigliani (1963) later developed the same concept. This hypothesis presents a basic understanding of the consumption and savings behavior of an individual over his/her lifetime. It also explains the motivations behind the participation of individuals in different pension systems, suggesting that they intend to optimize their utility by smoothing consumption over their lifetime, which means ensuring stable consumption through either PAYG or IA systems.

Based on this understanding of individual motivations, McFadden (1974) proposed the Discrete Choice Model, which presents an important analytical tool for further investigating the selection process of the pension system. This model deals with a detailed analysis of how individuals examine various attributes of each system, including risk, expected return, and replacement rates, within their socioeconomic environment context. This model is flexible and incorporates different variables like income levels, risk aversion, expected longevity, etc., making it particularly adept at calculating how individuals make their pension choices.

Along the same line, the research of Barr and Diamond (2006) on mixed pension systems adds some complexities to the understanding of the framework. Their analysis explores the trade-offs between PAYG and IA systems, supporting a mixed system approach that balances the strengths of both systems while focusing on policy objectives like poverty alleviation, consumption smoothing, and redistribution. Their emphasis on the implications of these trade-offs provides a refined understanding of the potential outcomes of different pension system designs.

In this sense, it should be mentioned that simulation techniques, such as Monte Carlo simulations, are important in studying pension systems because they serve as an integral tool to simulate different scenarios. Some of the earlier simulation models employed are the dynamic simulation model (Bütler & Teppa, 2007) and overlapping generations models (Heijdra & Romp, 2009). These models provide an important understanding of how individuals might behave under different pension arrangements and how demographic trends could influence the stability and popularity of PAYG versus IA systems. These simulation studies are particularly relevant in countries like Colombia, where demographic shifts and economic changes are ongoing. A recent systematic review of portfolio optimization models for pension purposes highlights methodological gaps and emerging trends in pension fund management (Moreira, dos Santos, & Gonzalez, 2025).

While applying these theoretical models in the Colombian context, it is important to consider the specific demographic, economic, and institutional characteristics that affect pension system preferences and outcomes. For example, Rofman et al. (2013) discuss the challenges and opportunities in Latin America, with a focus on the impact of demographic changes, labor market structure, and fiscal constraints on pension systems. Their analysis provides an empirical

framework for understanding how individual and systemic factors come together to influence pension choices in Colombia. Thus, the various theoretical models and empirical research presented in this section propose to create a comprehensive, robust understanding of the factors influencing pension system choice. It integrates life-cycle hypotheses, behavioral insights, risk and welfare considerations, empirical evidence, and simulation-based projections to provide a comprehensive view of the individual decision-making process in the pension selection process.

## II. Model

It is assumed that there may be  $N$  individuals who decide on which pension system to join at the beginning of their working life; at that point, each individual knows with certainty the values of all relevant variables. The individual's utility of being in a pension system  $\in \{1,0\}$ , where 1 denotes the Pay-as-you-go (PAYG) pension systems and 0 denotes the individual account pension systems (IA), is given by the following static utility function

$$U_j^i(C, R_j, P_j^i) = \ln(C^i) - \frac{\gamma}{2} \frac{(R_j - P_j^i)^2}{(P_j^i)^2}, \quad (1)$$

where:  $U^i$  is the utility function that measures the satisfaction of individual  $i$ ,  $C^i$  an aggregation variable that represents all the variables that affect the satisfaction of individual  $i$  before retirement, which for simplicity we will simply denote as consumption and which are the same regardless of the pension system where the individual is affiliated,  $R_j \in [0,1]$  is the replacement rate that system  $j$  delivers, that is, the proportion of pre-retirement income that is expected to be received in the form of a pension; it is assumed that this variable is known with certainty,  $\gamma$  is a parameter that measures the individual's risk aversion, finally  $P_j^i \in [0,1]$  is the probability of receiving a pension, that is, of reaching the minimum contributions required to retire in pension system  $j$ . This probability is understood by the individual as the ratio weeks contributed  $n_i$  - number of contributions required by system  $j$  ( $\alpha_j$ ), which represents the minimum number of contributions required to retire in system  $j$ . Thus, the function that determines the probability of receiving the pension is given by:

$$P_j^i(n_i) = \frac{\min\{n_i, \alpha_j\}}{\alpha_j}. \quad (2)$$

This utility function can be seen as a combination of two well-established utility forms. The first term,  $\ln(C^i)$  is a logarithmic utility over consumption, a common assumption in economics to represent risk-averse preferences (Arrow, 1971; Pratt, 1978). The second term represents a quadratic loss function related to the difference between the expected and realized replacement rate, scaled by the probability of receiving a pension. This loss term reflects aversion to uncertainty regarding pension outcomes and can be likened to models that incorporate quadratic utility or loss aversion (Friedman & Savage, 1948; Kahneman & Tversky, 1979).

Remember that, at the time of making his decision, the individual knows with certainty how many weeks he will be able to contribute throughout his working life, what his pre-retirement consumption level will be, the replacement rate of each system, and the minimum number of contributions required to retire in each system. Following the Additive Random Utility Model

(ARUM) proposed in the discrete choice literature (Train, 2009; McFadden, 1974), individual  $i$  will optimally select either a Pay-as-you-go (PAYG) pension system ( $j^{i,*} = 1$ ) or an individual account pension system (IA) ( $j^{i,*} = 0$ ), if its utility is greater than that of the alternative system.<sup>2</sup> So the selection mechanism can be summarized in the following expression, similar to the choice mechanisms discussed by Ben-Akiva and Lerman (1985):

$$j^{i,*} = \begin{cases} 0, & U_1^i(C^i, R_1, P_1^i) - U_0^i(C^i, R_0, P_0^i) < 0 \\ 1, & U_1^i(C^i, R_1, P_1^i) - U_0^i(C^i, R_0, P_0^i) \geq 0 \end{cases} . \quad (3)$$

It is assumed that each individual is assigned to a social scale at birth and cannot change it throughout his or her life. This assignment is made via real wages upon entering the labor market. This assumption is supported by studies that have documented the lack of social mobility and the persistence of intergenerational inequality in different contexts (Chetty et al., 2014; Piketty, 2014; Corak, 2013). The mechanism of Martínez et al. (2022) is followed, so that individuals enter the labor market at age  $T_I$ , while the retirement age is  $T_R$ . The real wage of individual  $i$  is denoted by  $w^i$  which remains constant throughout his life. This real wage is randomly assigned from a log-normal distribution of the form

$$f_{w^i}(w^i; \mu, \sigma) = \frac{1}{w^i \sigma \sqrt{2\pi}} \exp \left\{ -\frac{(\log w^i - \mu)^2}{2\sigma^2} \right\}, \quad w^i > 1. \quad (4)$$

At each time  $t$  in which the individual is active in the labor market, he may be formally working and contributing ( $d_t^i = 1$ ) or he may not be contributing because he is unemployed or working in the informal sector ( $d_t^i = 0$ ). This variable is described by the following Bernoulli process.

$$d_t^i = \begin{cases} 0, & \pi^i \geq \frac{\log(w^i)}{\log(\text{Max } \mathbf{w})} \\ 1, & \pi^i < \frac{\log(w^i)}{\log(\text{Max } \mathbf{w})} \end{cases}, \quad (5)$$

where  $\mathbf{w}$  is a vector of dimension,  $N$  times 1 containing the wages of all individuals. The probability of contributing to the pension system depends on relative income inequality, so that in each period the poorer the individual is in relative terms, the less likely he/she is to contribute to the pension system and therefore the less likely he/she is to reach the necessary contributions to retire (Martínez et al., 2022). Remember that  $n_i$  is the number of times individual  $i$  contributes to the pension system. In each period, the determination of whether the individual contributes or not is given by the variable  $d_t^i$  which is an independent Bernoulli random variable in time that depends on the probability  $\pi^i$ , i.e.,  $d_t^i \sim B(\pi^i)$ . We know that in each time period, its expected value is  $E[d_t^i] = \pi^i$  and its variance is given by  $V[d_t^i] = \pi^i(1 - \pi^i)$ . The individual will be in the labor market with the possibility to contribute between periods  $T_I$  and  $T_R$ . So, the number of total contributions that individual  $i$  will be able to accumulate over his lifetime follows a binomial distribution of the form  $n_i \sim \text{Bin}(T_R - T_I, \pi^i)$ . This is why the number of total contributions that each individual expects to achieve will be  $E[n_i] = \pi^i(T_R - T_I)$ . Thus,  $\pi^i$  is nothing more than the contribution density of individual  $i$ . This information is known by the individual at the beginning

<sup>2</sup> To assess how behavioral frictions (such as decision noise and systematic biases) modify these predictions, we develop a bounded-rationality extension in Appendix B.

of his working life, that is, at the moment of choosing the pension system. Thus, the model presents the following results.

**Result 1:** If the individual has the same probability of retiring in both pension systems, he selects the system that gives him the highest replacement rate.

**Proof:** The individual selects the public system  $j^{i,*} = 1$  si  $U_1^i(C, R_j^i, P_j^i) - U_0^i(C, R_j^i, P_j^i) \geq 0$ , substituting the functional form we have that this condition is summed up to  $\ln(C^i) - \frac{\gamma(R_1 - P_1^i)^2}{2(P_1^i)^2} \geq \ln(C^i) - \frac{\gamma(R_0 - P_0^i)^2}{2(P_0^i)^2}$ . Since the probability of retirement and the level of consumption prior to retirement are assumed to be equal in both systems, this expression reduces to  $R_1 \geq R_0$ , i.e., the individual selects the public system if its replacement rate is higher. Similarly, we find that the individual selects the private  $j^{i,*} = 0$  if  $R_1 < R_0$ .

**Result 2:** The utility function is increasing in the replacement rate when the replacement rate is less than the probability of receiving the pension and decreasing in the replacement rate otherwise.

**Proof:** Deriving the utility function shown in equation (1) with respect to the replacement rate, it is found that

$$\frac{dU_j^i(C^i, R_j, P_j^i)}{dR_j} = \begin{cases} -\gamma \frac{(R_j - P_j^i)}{(P_j^i)^2} > 0, & R_j < P_j^i \\ -\gamma \frac{(R_j - P_j^i)}{(P_j^i)^2} < 0, & R_j > P_j^i \end{cases} \quad (6)$$

From result 2, we conclude that if the individual perceives the probability of receiving the pension to be high (relative to the replacement rate), then increases the high replacement rate to generate higher utility. On the other hand, if the probability of receiving the pension is low (relative to the replacement rate), then increases in the replacement rate generate disutility to the individual. This can be interpreted separately for each pension system. In the public system, a high replacement rate is paid by taxes that reduce the individual's consumption, so if the individual does not benefit from the high replacement rate, they only pay the costs. In the case of the private, individually funded system, arguments linked to envy can be used to show that a high replacement rate can generate disutility for individuals when they will not benefit from it because it makes them poorer in terms of their pre-retirement situation compared to those who will be able to retire.

**Result 3:** The utility function is increasing with respect to the probability of receiving the pension when the replacement rate is positive and less than the probability of receiving the pension. However, it remains unaffected by changes in the probability of receiving the pension if the replacement rate is zero.

**Proof:** Deriving the utility function shown in equation (1) with respect to the probability of receiving the pension, it is found that

$$\frac{dU_j^i(C^i, R_j, P_j^i)}{dP_j^i} = \frac{\gamma R_j (R_j - P_j^i)}{(P_j^i)^3} \quad (7)$$

In Equation (5)  $\gamma > 0$ ,  $(P_j^i)^3 > 0$  and  $R_j \geq P_j^i$ . If  $R_j = 0 \Rightarrow \frac{dU_j^i(c^i, R_j, P_j^i)}{dP_j^i} = 0$  Utility does not change when  $P_j^i$  varies; hence it is invariant to  $P_j^i$ . If  $R_j > 0$  the sign depends on  $(R_j - P_j^i)$ . If  $R_j > P_j^i$ , then  $(R_j - P_j^i) > 0$ , and  $\frac{dU_j^i(c^i, R_j, P_j^i)}{dP_j^i} > 0$ , therefore the utility increases with  $P_j^i$ . If  $R_j = P_j^i$ , then  $(R_j - P_j^i) = 0$ , and  $\frac{dU_j^i(c^i, R_j, P_j^i)}{dP_j^i} = 0$ , therefore at this point, utility reaches a local maximum. The case  $R_j < P_j^i$  is not possible by assumption.

That is, individuals always have utility derived from increasing their probability of obtaining a pension unless the replacement rate is zero, in which case the individual does not obtain any utility from increasing the probability of retiring. This last case can be interpreted as the scenario where the individual thinks that the system will not provide him with any pension; this risk is common in pension systems where pension funds are used in a discretionary manner by the government for purposes other than those for which they are intended.

**Result 4:** Greater risk aversion ( $\gamma$ ) decreases an individual's utility whenever there is a mismatch between the replacement rate  $R_j$  and the probability of receiving a pension  $P_j^i$ . Conversely, if  $R_j$  equals  $P_j^i$ , changes in  $\gamma$  have no effect on utility.

**Proof:** Differentiate equation (1) with Respect to  $\gamma$

$$\frac{dU_j^i(c^i, R_j, P_j^i)}{dP_j^i} = -\frac{1}{2} \frac{(R_j - P_j^i)^2}{(P_j^i)^2} \quad (8)$$

Notice that this expression does not depend on  $\gamma$  (other than the factor we factored out), which means the rate of change of utility with respect to  $\gamma$  is constant for given  $R_j$  and  $P_j^i$ . If  $R_j = P_j^i$ , then  $(R_j - P_j^i) = 0$ , and  $\frac{dU_j^i(c^i, R_j, P_j^i)}{d\gamma} = 0$ , therefore, changes in  $\gamma$  does not affect utility. If  $R_j \neq P_j^i$ , then  $(R_j - P_j^i)^2 > 0$  thus  $\frac{dU_j^i(c^i, R_j, P_j^i)}{d\gamma} < 0$ , The derivative is strictly negative, implying that as  $\gamma$  increases, utility decreases whenever  $(R_j - P_j^i)^2 > 0$ .

When there is a mismatch between an individual's probability of receiving a pension  $P_j^i$  and the replacement rate  $R_j$ , risk aversion amplifies the negative effect on utility because the individual perceives greater uncertainty or discrepancy in outcomes. As  $\gamma$  increases, the penalty for this mismatch grows, reflecting a heightened aversion to the possibility that the promised benefits may not align with what the individual expects to receive. By contrast, if  $R_j$  perfectly matches  $P_j^i$ , no mismatch exists, and changes in  $\gamma$  no longer affect utility because there is no uncertainty penalty to magnify.

Designing pension policies that minimize the gap between promised benefits and realistic probabilities of achieving those benefits is especially beneficial for risk-averse individuals. If the system ensures  $R_j \approx P_j^i$  highly risk-averse workers will not suffer added disutility from

misalignment. Thus, policies that enhance transparency, consistency, and credible commitment to a specific replacement rate can significantly reduce the utility costs associated with heightened risk aversion.

### A. Replacement rate, subsidies and surplus

The replacement rate of the IA ( $R_2$ ) is set according to the market interest rate  $r$ . For an individual who has contributed for  $n = T_R - T_I$  periods and has made contributions  $H_t^i = \tau w^i d_t^i$  in period  $t$ , where  $\tau$  is the individual's contribution rate to the pension system, assumed to be constant. The amount of savings accumulated in the IA system at retirement, considering a constant interest rate  $r$ , would be:

$$A^i = \tau w^i \sum_{t=T_I}^{T_R} d_t^i (1+r)^{T_R-t} \quad (9)$$

The replacement rate is the ratio of the pension to the individual's last salary. If we assume that the individual withdraws his savings as an annuity for  $M$  years (life expectancy after retirement), the annual pension would be:

$$H_1^i = \frac{A^i}{M} \quad (10)$$

The replacement rate  $R_1$  in the individually funded system would be:

$$R_1 = \frac{H_1^i}{w^i} \quad (11)$$

**Result 5:** Individuals with higher incomes will have a higher replacement rate because their contribution densities are higher.

**Proof:** The replacement rate of the IA ( $R_1$ ) can be rewritten by joining equations 9, 10, and 11 as

$$R_1 = \frac{H_1^i}{w^i} = \frac{\frac{A^i}{M}}{w^i} = \frac{\tau w^i \sum_{t=T_I}^{T_R} d_t^i (1+r)^{T_R-t}}{M w^i} = \frac{\tau}{M} \sum_{t=T_I}^{T_R} d_t^i (1+r)^{T_R-t} \quad (12)$$

Please note that  $\frac{\partial d_t^i}{\partial w^i} \geq 0$ , so from (11) it can be deduced that  $\frac{\partial R_1}{\partial w^i} \geq 0$ .

In the IA system, there is a direct correspondence between what the individual saves and what he receives, so the individual can only increase his pension by contributing more money or contributing on more occasions. On the other hand, the replacement rate of the PAYG system is exogenous and does not depend on the market interest rate. This implies that, if the PAYG system grants a higher replacement rate than the IA system to the individual  $i$ , i.e.  $R_0 > R_1$ , then his pension is being subsidized, the amount of subsidy in each period of time is given by the expression:

$$s_i = (R_0 - R_1)w^i . \quad (13)$$

A tax is defined as regressive if the tax rate decreases as income increases. On this basis, a subsidy is regressive if it is higher as the individual's income is higher, i.e., if it

$$\frac{\partial s_i}{\partial w^i} \geq 0 . \quad (14)$$

In this case, equation (13) shows that the higher the replacement rate of the PAYG system, the more regressive the subsidy is. Table 2 shows that the replacement rate of the PAYG system is 73.1% while that of the IA system is 38.6%, so the PAYG system generates a subsidy that is very regressive, i.e., it very markedly benefits higher-income individuals. Because this subsidy is financed with public resources and because Colombia's distributive system is very regressive (Castañeda et al., 2020; Castañeda & Doyle, 2019), this regressivity of the subsidy is not only benefiting higher-income individuals, but it is also extracting those resources from lower-income individuals.

### ***B. PAYG for High Earners and IA for Low Earners***

Individuals who do not meet the minimum contribution requirements to qualify for a pension receive their accumulated contributions differently depending on the system. In the PAYG system, individuals receive the total amount of their contributions adjusted solely by inflation, using the Consumer Price Index (CPI). In contrast, in the IA system, managed by private funds, individuals are refunded the balance of their individual accounts, which includes not only inflation-adjusted contributions but also the real returns generated by the investments made by the fund. As a result, refunds under the IA system ( $L_1$ ) are typically higher than those in the PAYG system ( $L_0$ ).

To formalize this, we denote  $L_0$  as the refund under the PAYG system, which is the sum of contributions adjusted by inflation, and  $L_1$  as the refund under the IA system, which includes contributions, inflation adjustments, and real investment returns. By construction,  $L_1 > L_0$  reflecting the additional returns generated in the IA system (López & Sarmiento, 2019).

This mechanism is important when analyzing individual preferences for pension systems, particularly for those who fail to qualify for a pension. Individuals face two potential outcomes in each system. With a probability  $P_j^i$ , they qualify for a pension and receive a  $R_j w^i$ , where  $R_j$  is the replacement rate and  $w^i$  is their wage. Alternatively, with probability  $1 - P_j^i$ , they do not qualify and receive the refund  $L_j$ , which depends on the system they are in. The higher replacement rate in the PAYG system ( $R_0 > R_1$ ) makes it more attractive for individuals likely to qualify for a pension, while the larger refund in the IA system ( $L_1 > L_0$ ) appeals to those with a low probability of qualification.

In this section, we assume that  $R_0 > R_1$  and  $L_1 > L_0$ , consistent with the observed characteristics of the pension systems in Colombia. These assumptions underpin the analysis of the wage threshold  $w^*$  that determines system preference, as shown in the result and proof presented below.

**Result 6:** There is a critical wage  $w^*$  such that individuals with wages  $w^i \geq w^*$  prefer the PAYG system, whereas those with wages  $w^i < w^*$  prefer the individually funded (IA) system.

**Proof:** Let each individual face two possible outcomes under system  $j \in \{0,1\}$ . The individual is entitled to a pension with probability  $P_j^i$ , yielding a pension  $R_j w^i$ . The Lump sum Refund: Occurs with probability  $1 - P_j^i$ . Denote the lump sum in PAYG by  $L_0$  (inflation-adjusted contributions) and that in IA by  $L_1$  (contributions plus real returns).

Given  $R_0 > R_1$  and  $L_1 > L_0$ , we compare two utilities  $\Delta U^i(w^i) = U_1^i(C^i, R_1(w^i), P_1^i(w^i)) - U_0^i(C^i, R_0(w^i), P_0^i(w^i))$ . If  $w^i < w^*$  They struggle more to meet the required contributions. Hence, they face a relatively large probability of not qualifying for a pension  $1 - P_j^i$ . Because the IA system grants a bigger lump sum refund  $L_1 > L_0$  in that failure state, they prefer IA even if its replacement rate  $R_1$  is lower. If  $w^i \geq w^*$  They tend to make sufficient contributions to qualify for a pension. In this case, the higher replacement rate  $R_0$  in PAYG dominates. Therefore, they prefer PAYG since they will likely retire with a higher pension.

Note that  $\Delta U^i(w^i)$  depends on  $P_0^i(w^i)$  and  $P_1^i(w^i)$ , the probabilities of qualifying in each system, both increasing in  $w^i$  but with different slopes/thresholds. Also, the difference in lump sum refunds ( $L_1 - L_0$ ) and replacement rates ( $R_1 - R_0$ ). As  $w^i$  rises, the advantage of receiving the (larger) replacement rate  $R_0$  in PAYG eventually outweighs the advantage of the superior lump sum in IA when the chance of pension qualification becomes very high. By continuity, there exists a wage  $w^*$  such that:

$$\Delta U^i(w^*) = 0 \implies \begin{cases} w^i < w^* \rightarrow \Delta U^i(w^i) < 0 \\ w^i > w^* \rightarrow \Delta U^i(w^i) > 0 \end{cases}$$

Hence, a wage threshold emerges that separates those better off in the IA system (low-wage individuals more likely to fail reaching the pension but getting a larger lump-sum) from those better off in the PAYG system (high-wage individuals who qualify for a higher replacement rate).

Low earners, facing a high probability of not qualifying for a pension, are drawn to the more generous refund in the IA system. Meanwhile, high earners, who can reliably accumulate the required contributions, gravitate toward the PAYG system, which offers a higher replacement rate  $R_0$ . This interplay of lump sum payback versus pension payoff naturally produces a switching point  $w^*$  in the wage distribution.

Policymakers should note that raising or lowering the required contributions or adjusting refund rules can shift the wage threshold. If the public system requires fewer contributions but only refunds inflation-adjusted sums, it appeals more to lower earners if that threshold is not too high. Conversely, a higher guaranteed replacement rate in the PAYG system mainly benefits those with a strong likelihood of pension eligibility, i.e., higher earners. Understanding this threshold can help target reforms aimed at achieving better coverage, equity, and sustainability in pension systems.

**Result 7:** If the difference ( $L_1 - L_0$ ) increases, meaning that the IA system's lump sum refund becomes more generous relative to the PAYG system, then the critical wage  $w^*$  at which individuals switch from preferring one system to the other increases. Equivalently, more individuals (those with wages up to this now higher  $w^*$ ) will prefer the IA system due to the larger refund advantage if they fail to qualify for a pension.

**Proof:** If the individual fails to qualify for a pension in system  $j$  (which occurs with probability  $1 - P_j^i$ ), they receive a lump sum  $L_j$ . As a result, the utility in each system is shaped by both the pension payoff and the lump sum payoff. Because  $\Delta U^i(w^i)$  is continuous in  $w^i$ , there exists a critical wage  $w^*$  such that  $\Delta U^i(w^*) = 0$ , dividing individuals who prefer IA ( $\Delta U^i(w^i) > 0$ ) from those who prefer PAYG ( $\Delta U^i(w^i) < 0$ ). The portion of  $\Delta U^i(w^i)$  corresponding to the “no-pension” outcome grows in favor of PAYG. For a given  $w^i$  near the threshold, if  $\Delta U^i(w^i)$  was slightly below zero (PAYG was marginally preferred), the increase in  $\Delta L$  can make  $\Delta U^i(w^i)$  switch to above zero (IA now preferred). As a result, the new equilibrium wage  $w^{i'}$  solving  $U^i(w^{i'}) = 0$  must be higher than the old  $w^*$ . Hence, the threshold wage rises in response to a more generous IA lump-sum.

When the gap ( $L_1 - L_0$ ) widens IA becomes more appealing in the event of failing to qualify for a pension. Thus, individuals who are on the fence, i.e., whose wage  $w^i$  is at or around the original threshold may now switch to preferring IA because the worst-case scenario (no pension) yields a much better lump sum in the IA system. This shift effectively expands the lower and middle wage portion of the population for whom IA is optimal, pushing the threshold wage  $w^*$  upward.

Policymakers can see that increasing real returns or otherwise boosting the lump sum advantages in IA will pull more workers (including some with moderate wages) away from PAYG. On the one hand, it may reduce the population at risk of ending up with minimal or no returns in the public system if they fail to qualify. On the other hand, it can impact the solvency or membership base of the PAYG system, as those who might have contributed cross-subsidies in a high replacement-rate regime may migrate to IA. Similarly, a reduction in the refunds under the PAYG system ( $L_0$ ) would also drive migration from PAYG to IA.

### III. Monte Carlo Experiment

#### A. Calibration

In pay-as-you-go pension systems (Colpensiones), the replacement rate is established in Law 797/2003, according to which it must be between 55% and 80% depending on the weeks contributed and the individual's base settlement income. If the individual reaches the minimum weeks necessary to retire (1,300 contributed weeks), the replacement rate is 65% and increases by 1.5% for each additional 50 weeks until reaching a maximum replacement rate of 80%. However, article 34 of Law 100 of 1993 establishes that 65% of the replacement rate is penalized by the level of income in relation to the minimum wage, so the replacement rate is calculated from the form,  $R = 65.5 - 0.50 S$ , where  $S$  is the number of current minimum wages. On the other hand, the replacement rate in individual account (IA) pension systems varies depending on the capital accumulated by the worker and the profitability obtained by the fund. According to López and Sarmiento (2019), on average the pay-as-you-go pension systems provide a replacement rate of 73.1% and the IA 38.6%, these latter values are those used in the simulation exercise, that is,  $R_1 = 0.731$  and  $R_2 = 0.386$ .

For the parameters of the number of minimum contributions, it was previously mentioned that in the pay-as-you-go pension systems administered by Colpensiones, 1,300 weeks of contributions are required to access the pension, which is equivalent to around 26 years of contributions. In addition, a minimum age of 62 years for men and 57 years for women must be met. While in the IA pension systems administered by private pension funds, 1,150 weeks of contributions are required to access the pension, which is equivalent to around 22 years. In addition, a minimum age of 62 years for men and 57 for women must be met. That is,  $\alpha_1 = 1,300$  and  $\alpha_0 = 1,150$ .

The value of  $\gamma$  that measures the individual's risk aversion depends on several factors, such as income level, age, education, gender, and personal preferences. There is no value for this parameter in the literature on the Colombian economy. However, there are some studies that have estimated the risk aversion parameter for Colombia or for some sectors or population groups. Arango and Ramírez Hassan (2007) estimate an average value of 8.5 for the risk aversion parameter in Colombia, using the CCAPM model (Consumption Capital Asset Pricing Model) and macroeconomic data. Holt and Laury (2002) estimate an average value of 2.6 for the risk aversion parameter in a sample of Colombian university students, meanwhile Chicaíza et al. (2011) report values between 0.5 and 1.5 for the risk aversion parameter in Colombian doctors who participated in an experiment on clinical decisions. Based on this, a value of 1.5 is used, although in Appendix sensitivity exercises are carried out to demonstrate that variations in this parameter do not affect the results of the model.

In our context, the age of entry to the labor market is set at 16 years, which corresponds to the legal age in Colombia, while the retirement age is set at 62 years, corresponding to the maximum retirement age for men in Colombia. On the other hand, according to Azuero Zúñiga (2020) average contribution density in Colombia of 27.3%, i.e.,  $\bar{\pi}^i = \frac{1}{N} \sum_{i=1}^N \pi^i = 0.273$ . Thus, in order to calibrate the model to reflect economic reality, the parameters  $\mu$  and  $\sigma$  of the lognormal distribution are determined to make the model manage this average contribution density.

The procedure performed is that from these parameters  $\mu$  and  $\sigma$  we generate a sample  $\mathbf{w}$  of size  $N$  of wages. For each wage  $w_i$ , we calculate the ratio  $\pi_i$  with respect to the maximum wage in the sample, given by  $\pi_i = \frac{\log(w^i)}{\log(\text{Max } \mathbf{w})}$ , and ensure that this ratio is bounded between 0 and 1. The objective function we seek to minimize is  $f(\mu, \sigma) = |\frac{1}{N} \sum_{i=1}^N \pi_i - 0.27|$ . This function represents the discrepancy between the average wage ratio in our sample and the target of 27%. The optimization involves an exhaustive search over a predefined set of values for  $\mu$  and  $\sigma$ , selecting those that minimize  $f(\mu, \sigma)$  and, therefore, provide a more accurate representation of the Colombian economy. This procedure yields the values  $\mu = 0.502$  and  $\sigma = 0.396$ .<sup>3</sup>

**Table 1. Model Parameters**

Parameter	symbol	PAYG ( $\mathbf{j} = 1$ )	IA ( $\mathbf{j} = 0$ )
Replacement rate	$R_j$	0.731	0.386
Number of contributions required by the system	$\alpha_j$	1300	1150

<sup>3</sup> It is noted that the standard deviation used in the model,  $\sigma = 0.396$ , is the same as that reported by Hindriks & Myles (2013) for the distribution of talent in the population.

Mean of log normal distribution of wages	$\mu$	0.502	0.502
Standard deviation of the log-normal distribution of wages	$\sigma$	0.396	0.396
Age of entry into the Labor market.	$T_I$	16	16
Age of retirement of the contributor.	$T_R$	62	62
Risk aversion	$\gamma$	1.5	1.5
Average contribution density	$\bar{\pi}^i$	0.273	0.273

Source: Own elaboration.

## A. Model Output and Fits

Table 2 below shows that the model predicts that 65.7% of people will select individual account pension systems (IA) over pay-as-you-go (PAYG); this value is not statistically different from the 69% shown in the data (López & Sarmiento, 2019). Table 2 also shows that the model predicts that at the time of retirement, people on average will have contributed 557.6 weeks, while data from the Colombian pension system indicate that people contribute an average of 555 weeks when the time comes to retire (Azuero Zúñiga, 2020). The difference between both variables is not statistically significant, so the model correctly replicates the process. On the other hand, the data indicates that there is a positive and high correlation between the number of contributions and the individual's income level; the model gives the same result. However, the correlation found in the model is statistically greater than that shown in the data.

**Table 2. Model Parameters**

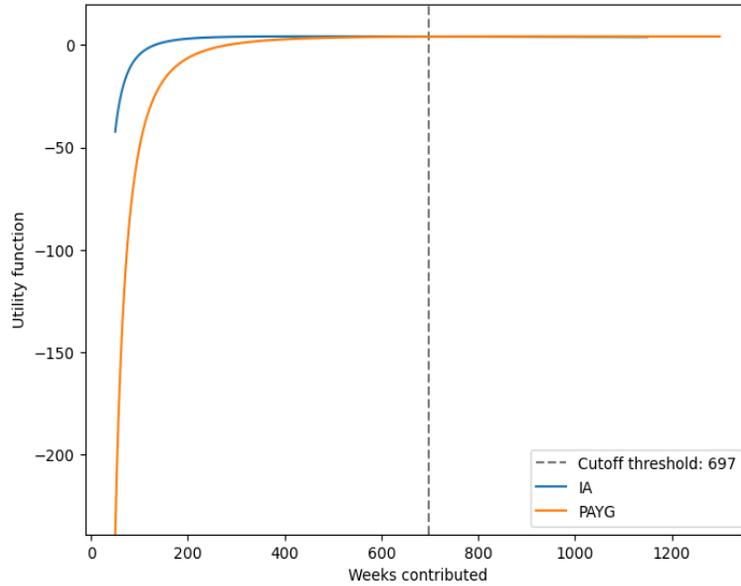
Parameter	Data	Model	Difference	Standard error	T-test
Individuals who select the IA over PAYG	69%	65.7%	-3.3	3.26	-1.01
Number of contributions upon reaching retirement age	555	557.6	2.6	33.56	0.08
Correlation ( $\mathbf{n}_i, \mathbf{w}^t$ )	0.71	0.95	0.24	0.0024	100

Source: Own elaboration.

## B. Simulations

As shown in Figure 1, under the parameters used we find that individuals select the Pay-as-you-go (PAYG) pension system over the individual account pension system (IA) if they believe that they will contribute more than  $E[n_i] = 697$  weeks throughout your working life. Therefore, 65.7% of people select the AI system over PAYG.

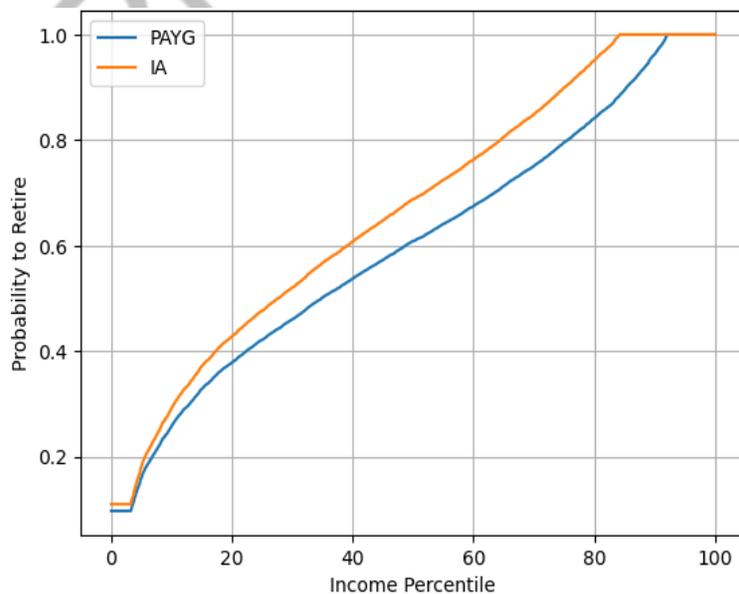
**Figure 1.** Utility levels of each pension system for each number of contributions made to the pension system



Source: Own elaboration.

There is a notable correlation between the number of contributions and the level of income, resulting in that, on average, individuals with higher incomes tend to opt for the pay-as-you-go (PAYG) system instead of the individual capitalization (IA) system. As evidenced in Figure 2, only the wealthiest 8% of the population have the same probability of pensioning in both the public and private sectors. For the other income percentiles, the possibility of obtaining a pension in the private sector exceeds that of the public sector, thus encouraging people with lower incomes to prefer the IA system, given that it is more accessible. Conversely, those with higher incomes are motivated to choose the PAYG system due to its higher replacement rate.

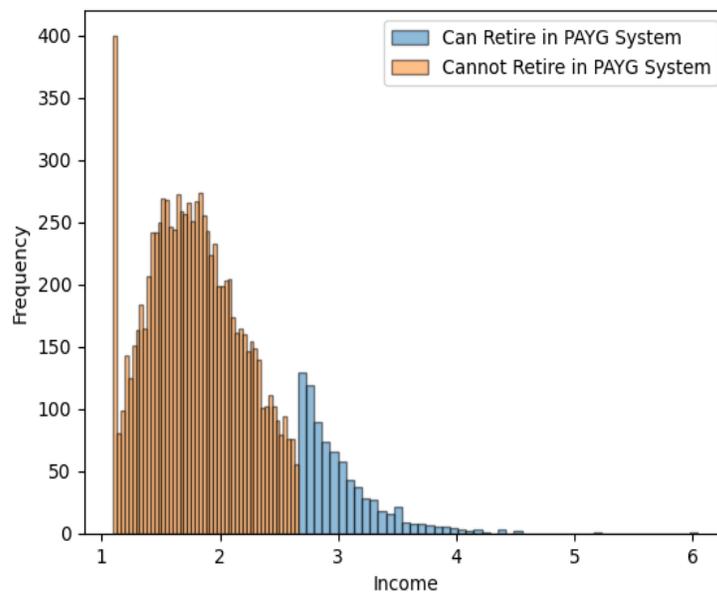
**Figure 2.** Probability of retirement in PAYG vs IA System



Source: Own elaboration.

Figure 3 presents a bifurcated distribution of income based on eligibility for retirement in the public pay-as-you-go (PAYG) system. The data reveals that only a minor fraction of the population, who receive the highest incomes, manage to qualify for withdrawal in the PAYG system, as evidenced by the blue bars. In contrast, most individuals, especially those with medium to low incomes, represented by the brown bars, do not qualify for retirement under PAYG. This pattern suggests a significant discrepancy in the distribution of pensions and highlights a possible preference or need of individuals with lower incomes for the individual capitalization (IA) system, which agrees with the simulations and previous results discussed, which indicate a pension system that differentially favors the highest income strata.

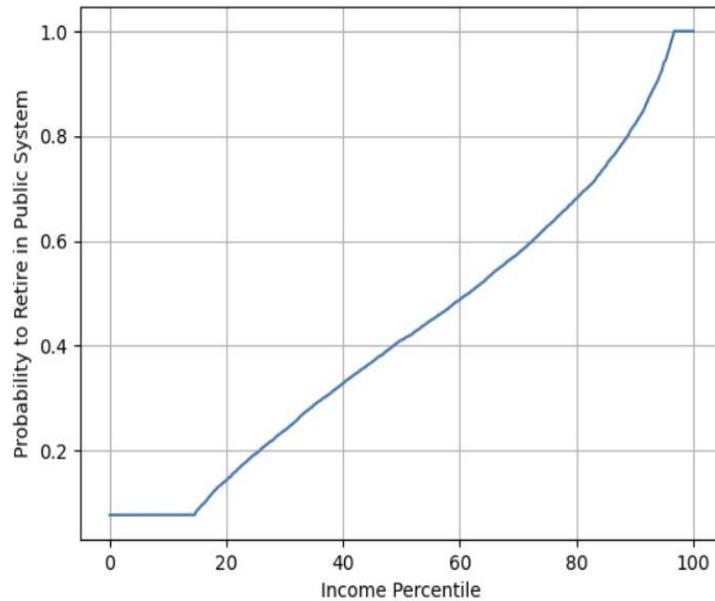
**Figure 3:** *Distribution of Incomes Based on Retirement in Public System (PAYG)*



Source: Own elaboration.

Figure 4 illustrates a growing and decidedly pronounced relationship between the probability of retiring in the public pension system and the individual's income percentile. Consistent with previous results, as we move up the income scale, the probability of qualifying for a pension in the public system increases substantially, converging towards certainty for the wealthiest. This upward curve highlights the disparity in access to pensions in the public system, which highlights a regressive dynamic within the Colombian pension framework: those with fewer resources are significantly less likely to benefit from the PAYG system compared to their counterparts in higher earnings. This phenomenon highlights the need for reforms that address equity within the pension system, a topic that is discussed in depth in the preceding sections of this work.

**Figure 4:** *Probability to Retire in Public System vs. Income Percentile*



*Source:* Own elaboration.

### Conclusions

This paper sets out to explore how individuals choose between pay-as-you-go (PAYG) and individual account (IA) pension systems, focusing on how replacement rates, contribution density, and wage levels affect retirement decisions given differences in replacement rates, contribution requirements, and lump sum refunds. The model demonstrates that there is a critical wage threshold above which individuals prefer the public PAYG system, due to its higher replacement rate, and below which individuals opt for IA, reflecting the IA system's more generous refunds for those who do not meet minimum contribution requirements.

Empirically, the model closely replicates key features of the Colombian pension context: the share of the population choosing IA approximates observed figures, and total contributions at retirement align with real-world data. The findings further indicate that the PAYG system disproportionately benefits higher earners through a regressive subsidy, whereas IA is more favorable for lower-income individuals who face a lower probability of qualifying for a pension. Moreover, heightened risk aversion amplifies the negative utility impact when promised replacement rates fail to match realistic probabilities of receiving a pension, underscoring the importance of aligning expectations with achievable outcomes.

Despite these insights, the analysis has certain limitations. First, wage mobility is assumed to be minimal, which may not fully capture real-life labor market dynamics. Second, the model treats future parameters (e.g., interest rates, contribution densities) as known and constant, possibly overlooking macroeconomic shocks. Third, heterogeneity in individual behavior, such as varied saving habits or differing degrees of pension system awareness, remains outside the scope. Fourth, our framework abstracts from Colombia's full labor-market heterogeneity by assuming all

individuals enter at age 16 and retire at age 62; drawing entry and exit ages from empirically observed distributions would yield more realistic contribution patterns and is left for future work.

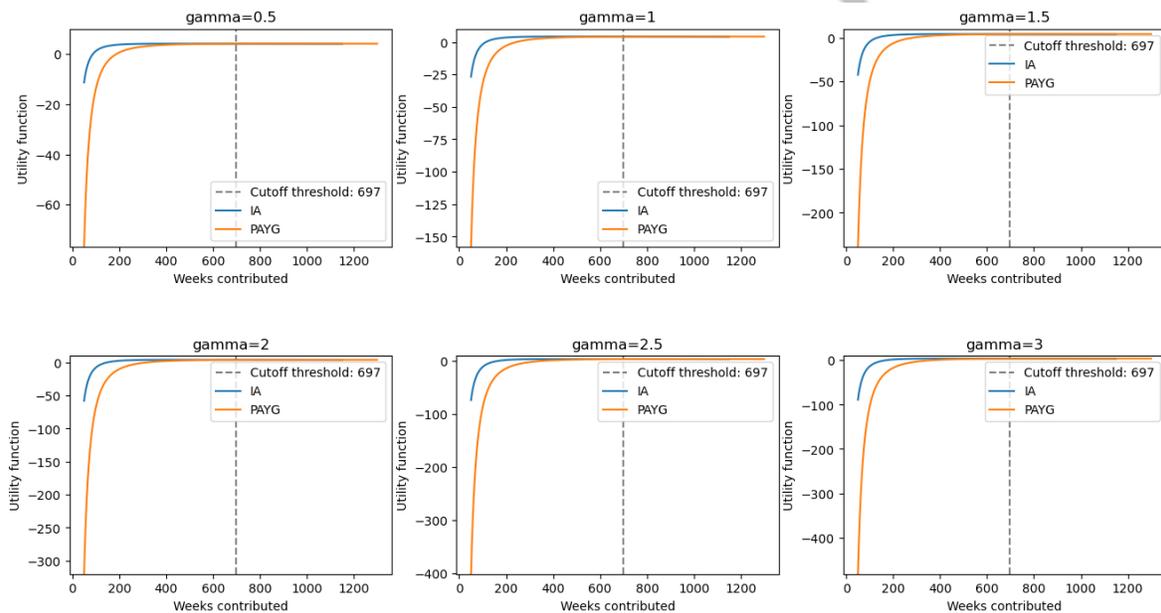
Future studies could enrich the framework by incorporating wage trajectories over time, stochastic interest rates, or additional behavioral factors, thereby refining the understanding of pension system preferences in diverse socioeconomic settings.

## Appendix

### 1. Sensitivity Analysis: Sensitivity to the value of risk aversion.

These values can serve as a reference or starting point to analyze the utility function proposed in this study. To test the robustness of the results, values of  $\gamma \in \{0.5, 1, 1.5, 2, 2.5, 3\}$  are taken. As mentioned before, the choice of one system over another is invariant to variations in this parameter.

**Figure A1.** Choice between public vs private pension system

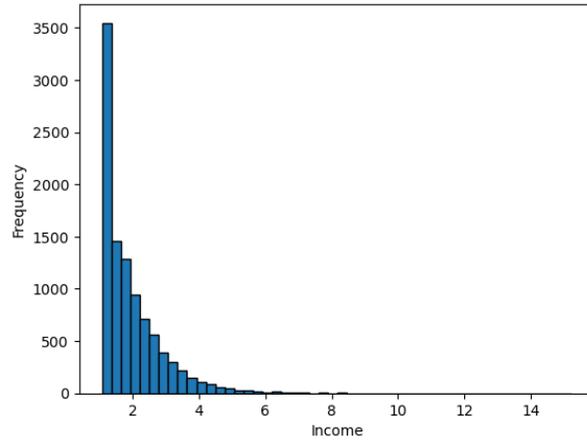


Source: Own elaboration.

### 2. Results

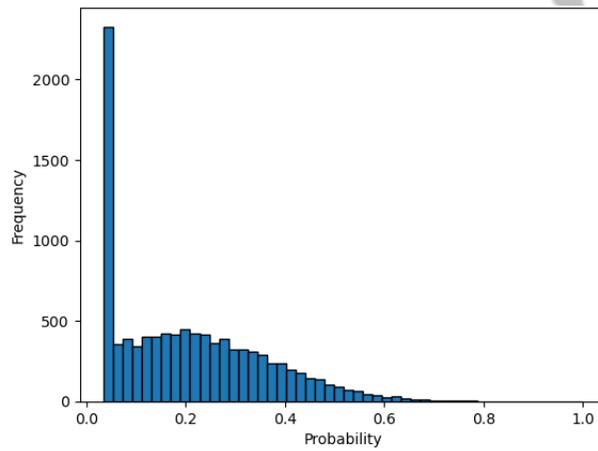
In this appendix, we present the results of the distributions of relevant variables generated from the Monte Carlo estimations, specifically a histogram of incomes, a histogram of probabilities to contribute to the pension system, and histogram of number of contributions.

**Figure A2.** Histogram of Incomes



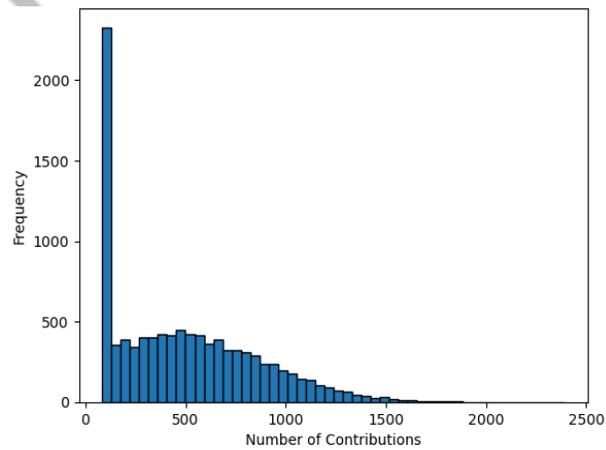
Source: Own elaboration.

**Figure A3.** Histogram of probabilities to contribute to pension system



Source: Own elaboration.

**Figure A4.** Histogram of number of contributions



Source: Own elaboration based on model simulations

### 3. Bounded Rationality Extension

In this appendix, we discuss a simple extension of the baseline discrete-choice model that allows for bounded rationality via both random decision noise and a systematic bias. We show analytically how each element affects the critical wage threshold and conclude that the core insight, high-wage workers favour PAYG, low-wage workers favour IA, remains intact.

We begin with the deterministic utility  $U_j^i(w^i)$  defined in equation (1), where  $j = 1$  denotes PAYG and  $j = 0$  denotes IA. We now assume that each individual makes choices based on a perceived utility

$$\widetilde{U}_j^i = U_j^i(w^i) - \delta_j + \varepsilon_j^i,$$

where  $\varepsilon_j^i \sim \text{EV1}(0, \lambda)$  are i.i.d. Type I extreme-value shocks (scale  $\lambda > 0$ ), capturing decision noise, and  $\delta_j$  is a systematic bias with  $\delta_1 = \delta > 0$  on PAYG and  $\delta_0 = 0$  on IA (representing, for example, switching costs, misperceived obstacles, or adviser steering). Under these assumptions, the probability that individual  $i$  chooses PAYG is the familiar logit formula:

$$\Pr(\text{PAYG} \mid w^i) = \frac{\exp((U_1^i(w^i) - \delta)/\lambda)}{\exp((U_1^i(w^i) - \delta)/\lambda) + \exp(U_0^i(w^i)/\lambda)}, \quad \text{B1}$$

and we denote  $\Delta U^i(w^i) \equiv U_1^i(w^i) - U_0^i(w^i)$ . Setting  $\Pr(\text{PAYG} \mid w^i) = 0.5$  in (B1) yields the indifference condition

$$\Delta U^i(w^i) = \delta. \quad \text{B2}$$

Since  $\Delta U^i(w^i)$  is strictly increasing in  $w^i$  (Result 6 in the main text), there is a unique solution  $w^{i*}(\delta)$  to (B2). Moreover, differentiating the log-odds  $\ln[\Pr/(1 - \Pr)]$  with respect to  $\Delta U^i$  shows that the steepness of the transition from IA to PAYG is  $1/\lambda$ : larger  $\lambda$  flattens the switch but does not move the 50% point.

**Result B1:** Let  $w^{i*}(0)$  be the deterministic threshold (the unique solution to  $\Delta U^i(w^i) = 0$ ). For any  $\delta > 0$ , there exists a unique  $w^*(\delta)$  solving  $\Delta U^i(w^{i*}(\delta)) = \delta$ , and

$$w^{i*}(\delta) > w^{i*}(0), \quad \frac{d w^{i*}(\delta)}{d\delta} = \frac{1}{\Delta U^{i'}(w^{i*}(\delta))} > 0.$$

**Proof.**

Define  $F(w^i, \delta) \equiv \Delta U^i(w^i) - \delta$ . By Result 6 we have  $\Delta U^{i'}(w^i) > 0$  for all  $w^i$ , so  $\partial F/\partial w^i = \Delta U^{i'}(w^i) > 0$  and  $\partial F/\partial \delta = -1$ . The Implicit Function Theorem then yields

$$\frac{d w^{i*}}{d\delta} = -\frac{\partial F/\partial \delta}{\partial F/\partial w^i} = -\frac{-1}{\Delta U^{i'}(w^{i*})} = \frac{1}{\Delta U^{i'}(w^{i*})} > 0.$$

Evaluating at  $\delta = 0$  recovers  $w^{i*}(0)$ , and monotonicity in  $\delta$  ensures  $w^{i*}(\delta) > w^{i*}(0)$  for  $\delta > 0$ .

Random decision noise ( $\lambda$ ) widens the wage-band in which individuals err but does *not* shift the median choice point. Systematic bias ( $\delta$ ) raises the median threshold  $w^{i*}$ , meaning that switching

costs, misinformation, or adviser steering lead some workers who would rationally prefer PAYG to choose IA. From a policy perspective, reducing  $\delta$  (through information campaigns, adviser regulation, or lower transaction costs) lowers  $w^{t*}$ , restoring PAYG to workers for whom it maximises expected utility, while reducing  $\lambda$  (via clearer defaults or behavioural “nudges”) narrows the zone of mis-choices without changing the median.

### Ethics Statement

This research article did not work with a person or groups of people to generate the data used in the methodology; therefore, it did not require the endorsement of an Ethics Committee for its realization.

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