Opportunities of Authentic Modelling Tasks in the Learning of Mathematical Modelling for Teaching. A Study With Prospective Mathematics Teachers

Oportunidades de las tareas auténticas de modelación en el aprendizaje de la modelización matemática para la enseñanza. Un estudio con futuros profesores de matemáticass

Oportunidades de tarefas autênticas de modelagem no aprendizado de modelagem matemática para o ensino. Um estudo com futuros professores de matemática

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Abstract

Teacher learning on mathematical modelling is a significant goal of mathematics education. This article presents the results of a study conducted within a course designed for prospective teachers. Rooted in the principles of authentic learning, authentic tasks were designed to promote the prospective teacher's learning of mathematical modelling and its teaching. Data were collected from 18 prospective teachers across two groups of a mathematical modelling course at a public University. Analysis of documents, videos, and interviews evidenced the learning achieved by prospective teachers about learning environment configuration, use of context, and modelling project development. The results of this study provide empirical evidence regarding characteristics and components that configure authentic tasks that contribute to mathematical modelling learning and its teaching. Some implications regarding the way to integrate authenticity in mathematical modelling emerged from this study.

Keywords:
Mathematical Modelling; Prospective Teacher Education; Authentic Modelling Tasks; Authenticity.

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Palabras clave:

modelación matemática; formación de futuros profesores; tareas auténticas de modelación; autenticidad

Resumen

El aprendizaje de la modelación matemática por parte de los profesores es un objetivo importante de la enseñanza de las matemáticas. Este artículo presenta los resultados de un estudio realizado en el marco de un curso diseñado para futuros profesores. Basándose en los principios del aprendizaje auténtico, se diseñaron tareas auténticas para promover el aprendizaje de la modelación matemática y su enseñanza por parte de los futuros profesores. Se recogieron datos de 18 futuros profesores en dos grupos de un curso de modelación matemática en una universidad pública en Colombia. El análisis de documentos, vídeos y entrevistas evidenciaron las oportunidades que ofrece este tipo de tareas para la configuración de ambientes de aprendizaje, el uso del contexto y el desarrollo de proyectos de modelación. Los resultados de este estudio aportan evidencias empíricas sobre las características y componentes que configuran las tareas auténticas que contribuyen al aprendizaje de la modelización matemática y a su enseñanza. De este estudio se desprenden algunas implicaciones sobre la forma de integrar la autenticidad en la modelización matemática.

Palabras Clave:

modelagem matemática; formação de futuros professores; tarefas autênticas de modelagem; autenticidade

Resumo

A aprendizagem de professores sobre modelagem matemática é uma meta importante da educação matemática. Este artigo apresenta os resultados de um estudo realizado em um curso criado para futuros professores. Com base nos princípios da aprendizagem autêntica, foram criadas tarefas autênticas para promover a aprendizagem de modelagem matemática e seu ensino por parte dos futuros professores. Foram coletados dados de 18 futuros professores em dois grupos de um curso de modelagem matemática em uma universidade pública na Colômbia. A análise de documentos, vídeos e entrevistas evidenciou as oportunidades para a configuração do ambiente de aprendizagem, o uso do contexto e o desenvolvimento de projetos de modelagem. Os resultados deste estudo fornecem evidências empíricas a respeito das características e dos componentes que configuram tarefas autênticas que contribuem para o aprendizado de modelagem matemática e seu ensino. Algumas implicações relacionadas à maneira de integrar a autenticidade na modelagem matemática surgiram deste estudo.

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1. Introduction

Mathematical modelling and its applications are considered a significant research domain of mathematics education (Blum et al., 2007). Models and modelling have been used in several social and scientific activities, but they also promote the development of mathematical thinking, student preparation for mathematical attitudes and competencies, participation in societal development, and integration of interdisciplinary curricula (Blum, 2011; Carmona-Mesa et al., 2020; Didis et al., 2016; Ocampo-Arenas & Parra-Zapata, 2022; Villa-Ochoa et al., 2022). Despite the relevance of mathematical modelling, its integration into curricula and everyday school life has become a challenge in many countries. For Kaiser (2014), how to integrate mathematical modelling into teaching and learning processes remains a point of discussion; various approaches are debated but there is not enough empirical evidence to support the effects of its integration into school practices.

Teachers play a fundamental role in the successful implementation of mathematical modelling in mathematics teaching (Blum et al., 2007; Villa-Ochoa, 2015). Therefore, first-hand experiences are necessary to promote the participation of prospective teachers as "modellers", and actions to use mathematical modelling in teaching (Blum et al., 2007; Villa-Ochoa et al., 2022; Widjaja, 2013). For Blum (2011) teachers should be prepared to support students and encourage multiple solutions. Likewise, teachers should encourage student strategies to solve modelling tasks and stimulate diverse meta-cognitive activities, in particular, reflections on solution processes and similarities between different situations and contexts. Strategies, resources, and learning environments remain an object of research in mathematical modelling in prospective mathematics teacher education.

Mathematical modelling seeks to transcend mathematics teacher preparation beyond routine practices in mathematics classes (Villa-Ochoa, 2015). Pragmatic and critical modelling approaches seek to ensure that students find useful mathematics, develop mathematical abilities, understand mathematical roles in society, have a better understanding of their environment, and practice responsible citizenship. For this purpose, teachers should encourage students to reproduce problems in which mathematics is applied, in contexts and areas where these happen in reality. These kinds of situations are authentic modelling problems (Kaiser & Schwarz, 2010). The design of authentic mathematical modelling tasks is a challenge in research (Greefrath & Vos, 2021; Niss, 2001; Villa-Ochoa, 2015).

Teachers should not only achieve a broad amalgam of knowledge that allows them to develop, design and implement modelling processes, but also knowledge about the possibilities these processes offer in school mathematics. For example, teachers should know about the cognitive demands of modelling tasks and management in class, and the resources and technology available for their development (Blum, 2011; Cetinkaya et al., 2016). Prospective teachers should also learn about modelling by experiencing a complete modelling process. It may allow teachers not only to focus on modelling as a teaching object, but it is also important that they have experiences in both modelling-as-content and modelling-as-vehicle perspectives (Julie & Mudaly, 2007; Villarreal et al., 2018).

Different aspects configure the teacher's knowledge for teaching mathematical modelling. It implies at least the integration of two characteristics: knowledge of mathematical modelling as a practice and knowledge of mathematical modelling teaching/learning

(Villa-Ochoa et al., 2021). On the one hand, the knowledge of mathematical modelling as a practice is related to the development of abilities to recognize, problems that require a solution in real situations, to make decisions about the techniques of constructing and validating models, to interpret results according to the previous needs of the context, among others. On the other hand, knowledge of mathematical modelling teaching/learning can make possible to create modelling tasks and opportunities for mathematical learning (modelling as vehicle) (Julie & Mudaly, 2007; Villa-Ochoa et al., 2021) and the mathematical modelling competences (mathematical modelling as a content) (Julie & Mudaly, 2007; Villa-Ochoa et al., 2021). In this sense, the challenge of teacher education programs is to provide experiences and opportunities that enable prospective teachers to learn mathematical modelling that integrates such characteristics.

In this vein, a course for prospective teachers was designed. In this course, they could engage in authentic modelling tasks for teaching mathematics in future, it means, they could participate in situations in Which They Were not Only Participating in mathematical modelling as a practice but also, recognized the affordances for teaching mathematics and orientations for their future teaching practices. To identify the contributions of these tasks, this article aims to answer this question: What opportunities do authentic tasks provide for learning mathematical modelling and its teaching for prospective teachers?

To answer this question, this article is organized into five sections. In the first, (section 2), we describe our theoretical background and narrow down the term "authentic modelling tasks for mathematics teachers". After that, we present the methodological design and the results. In the final section, we discuss the contributions of this study and conclude with final remarks and considerations for further research.

2. Theoretical Background

Adjectives such as 'authentic', 'relevant', 'real world', 'contextualised', and 'realistic' are pervasive in the literature on mathematics education. These adjectives refer to tasks that encourage and motivate students; however, evidence of these purposes is rarely presented (Beswick, 2011). According to Beswick (2011) "whereas 'real world' may describe word problems in which mathematics is presented in a simple sentence that provides minimal extra-mathematical information, 'authentic' and 'situated' tend to be used to convey something stronger" (p. 368-369). The call to include 'authenticity' in everyday school life is present not only in school mathematics but also is pervasive in the educational literature (Strobel et al., 2013). The debate on the meaning of 'authenticity', conditions and dimensions for its integration into the curricula has led to the development of theoretical perspectives and constructs. These have expanded in scope to encompass the learning of disciplines and resources in diverse environments (Barab et al., 2000; Herrington et al., 2010; Strobel et al., 2013). Some of considerations that guided the delimitation of an environment for prospective teachers learning of modelling are presented in this section.

2.1. Considerations About Authenticity in Learning Environments

The concept of authenticity was introduced simultaneously with a strong call for student-centred learning, and it is the hallmark of developing best practices for different learning environments (Herrington and Herrington 2006). Authentic learning environments have been designed to prepare prospective teachers for the usage of ICT (Barab et al., 2000; Herrington & Parker, 2013), and to know how these teachers integrate authenticity in designing mathematical problems and modelling (Almeida & Omodei, 2022; Cáceres et al., 2015).

Authentic environments can be assessed through different dimensions. Barab et al. (2000) distinguish three senses of authenticity:

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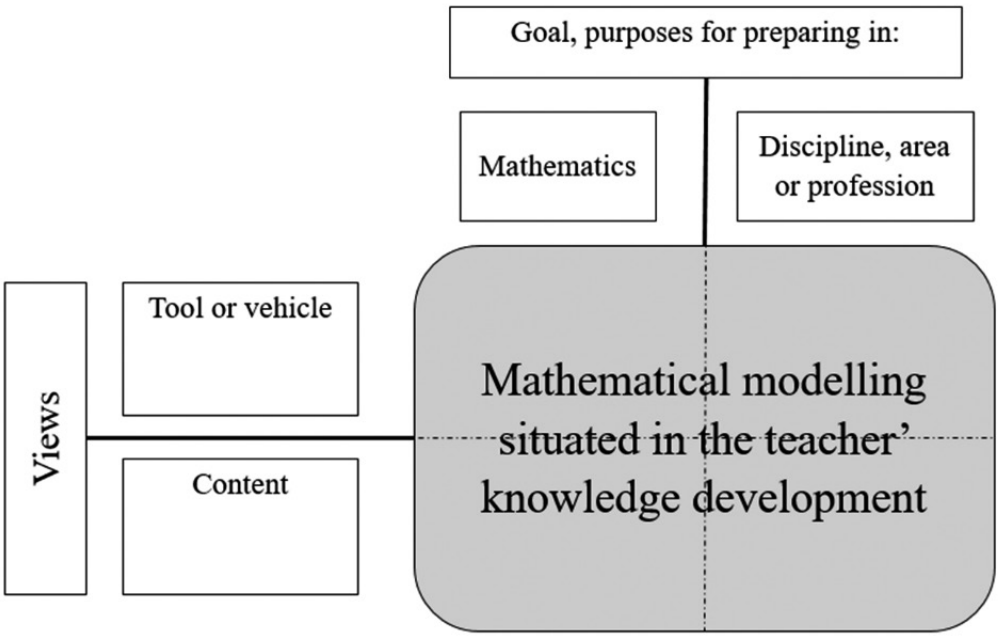
(a) authentic context – the context is like the real-world context (b) student activities are like real-world activities and (c) authenticity of impact - students' products are used in extracurricular activities. Strobel et al. (2013) present two additional dimensions: (a) "personal authenticity - projects are close to students' own life (i.e. life-stories of their neighbourhoods, biodiversity in the forest nearby)" and (b) "value authenticity - personal questions get answered or projects satisfy personal or community needs" (p. 144).

Despite the diversity of theoretical and empirical developments concerning the inclusion of authenticity in education, more research is needed to provide robust empirically derived models of authenticity (Strobel et al., 2013). In this sense, Herrington and Parker (Herrington & Parker, 2013) offer empirical evidence of the contributions of an authentic learning environment design in prospective teachers' preparation by using technologies as cognitive tools. The authors proposed a learning environment drawing on the principles of authentic learning proposed by Herrington et al. (2010). These principles are: (1) Provide authentic contexts that reflect the way knowledge will be used in real life, (2) Provide authentic activities, (3) Provide access to expert performances and the modelling of processes, (4) Provide multiple roles and perspectives, (5) Support collaborative construction of knowledge, (6) Promote reflection to enable the construction of abstractions, (7) Promote articulation to enable tacit knowledge to be made explicit, (8) Provide coaching and scaffolding by the teacher at critical times, (9) Provide authentic assessment of learning within the tasks.

2.2. *Prospective Teacher Learning Mathematical Modelling through Authentic Tasks*

The adjective authentic refers to the central role of the context, activities, practices, and their impact in learning because it involves physical and social conditions of the environment. These conditions represent an alternative learning where scientific contributions are explored. Based on the literature reviewed above, teacher education in mathematical modelling should not only address mathematical content but also focus on issues related to the teaching and learning of mathematics. In the case of mathematical modelling, this includes understanding modelling as both a vehicle for learning and as content itself (Villa-Ochoa et al., 2021). The following diagram illustrates these relations.

Fig 1. Representation of a teacher's modelling knowledge approach (Villa-Ochoa et al., 2021).



Taking into consideration these purposes, prospective teachers should learn about mathematics and its teaching through modelling. According to the characteristics of authentic learning (Herrington et al. 2010), mathematical and pedagogical learning is promoted when prospective teachers take part in experiences designed during the course. Learning about other phenomena and contexts was also promotedIn this approach, modelling learning focuses on its recognition as a vehicle to promote a different type of learning (Julie & Mudaly, 2007). Additionally, teachers learn to model mathematically, that means, to do mathematical modelling. In this approach, modelling is conceived as content (Julie & Mudaly, 2007), which means, it is treated as the object of study. The purpose is to learn about different processes, techniques, scope, and intentions of models and modelling, as well as considerations and pedagogical orientations for its future integration into the classroom.

In this sense, we use the term "authentic modelling tasks for mathematics teachers" to refer to tasks that simultaneously have a dual purpose—supporting the development of knowledge about mathematical modelling and its teaching—we use the term dual tasks. These tasks promote the learning of mathematical concepts related to modelling and help prospective teachers to reflect on the most effective pedagogical methods for teaching those concepts. These authentic modelling tasks strengthen mathematical modelling understanding and the use of mathematical modelling in teaching.

Regarding mathematical comprehension, this term grasps considerations from previous studies. The research literature informs that authentic mathematical tasks enhance mathematical problem-solving in students who have mathematical abilities and strong reading skills (Vicente & Manchado, 2016), but students' interest may decrease if the level of task difficulty is controlled (Rellensmann et al., 2017). Beyond motivation and performance, authenticity has been valued in mathematical problems and mathematical modelling, due to utilitarian goals. This puts real-world problem-solving, understanding of the real world, and the promotion of modelling competencies in the foreground (Kaiser & Schwarz, 2010).

Regarding knowledge about teaching, the use of authentic modelling tasks as a tool for teacher learning provides reliable classroom teaching scenarios as potential and credible experiences of what occurs in a mathematics class. These tasks should promote reflection on future teaching practices. They also provide important considerations about the participation of the teacher, the students, and their interactions in determining the phenomenon, the impact on both school and extracurricular practices, decisions about which variables and quantities to study, and the mathematical techniques and objects to employ.

3. Methodology

This study deals with the opportunities that authentic modelling tasks offer prospective mathematic teachers. To achieve the purpose, the focus was on the processes developed by prospective teachers during authentic modelling tasks. Therefore, a qualitative methodology was chosen. It allowed observing how authentic modelling tasks gave meaning to mathematical modelling and its teaching. For Bogdan and Biklen (2007), qualitative research is naturalistic, and therefore the context and environment of the phenomena are important in the construction of knowledge. In addition, the emphasis of qualitative research is on processes developed by the participants rather than on their products.

3.1. The Course and Participants

A mathematical modelling course for prospective teachers has been developed since 2013 as part of a bachelor's program offered by a school of Education at a public university in Medellín, Colombia. The course is distributed in 16 sessions of 4 hours each. As aforementioned, the authentic modelling tasks are provided by two approaches, first for participating in mathematical modelling practices and second for developing knowledge about mathematical modelling teaching.

The course is divided into four broad topics. In the first topic, modelling, perspectives, trends,

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and challenges for Mathematics Education were discussed. Some perspectives (Doerr & Pratt, 2008; Kaiser, 2017) about teaching practices regarding mathematical modelling, and implications for its integration in the classroom were studied. In the second, the contexts and classroom problems were researched as well as contextual and cultural roles in mathematical modelling, and the aspects involved in choosing a context for the class (e.g., Villa-Ochoa & Berrío, 2015). In the third, modelling was studied as process, activity, strategy, method, environment, practice, among others. Different understandings of mathematical modelling in school contexts and their influence on pedagogical or didactic practices were discussed. Finally, in the fourth topic, modelling with technology was studied. It was possible to discuss the advantages and limitations of technologies in the process of mathematical modelling (e.g., Molina-Toro et al., 2019).

The participants of this research were 18 prospective teachers divided into two groups in different semesters (Group A - 12 participants, and Group B – 6 participants). The prospective teachers participated voluntarily and were informed of the ethical protocol. Their names have been replaced by pseudonyms in this article.

3.2 Design and Implementation of Teaching Experiments

Teaching experiments are a type of intervention-based methodology used in research on mathematics education to understand how students learn (Bernabeu-Martínez et al., 2019). These experiments seek two objectives (a) improving teaching practice through didactic engineering approaches, acting on teaching-learning problems, and (b) increasing the theoretical understanding of the context related to these problems (Stylianides & Stylianides, 2013). Authentic modelling tasks for prospective teachers were designed based on the theoretical background (section 2). These tasks were conducted during the course, focusing on teaching and learning problems, and offering solutions based on theory.

As aforementioned, the authenticity in education deals with the interests, performances and visions of students and teachers. It is considered key for personal and value authenticity dimensions (Strobel et al., 2013). The authentic modelling tasks for prospective teachers were designed using the following characteristics:

- Promote modelling processes linked to phenomena of everyday life where the experiences, interests and needs of the participants were involved. Mathematical modelling projects were conducted during the semester with continuous advice from the teacher educators (teacher in the course), more details of the methodology of these projects were reported by Villa-Ochoa et al. (2021). The development of projects was linked to systematic techniques, mathematical language, and the identification of affordances for teaching. This aspect was configured considering the personal and value authenticity dimensions (Strobel et al., 2013), the principles of articulation (real and mathematical contexts), and authentic tasks (Herrington et al., 2010). For the project development, they met experts in each topic. According to each case, the experts supported the delimitation, the model construction, or the way to validate the results and interpretations. In this way, two principles proposed by Herrington et al. (2010) were also integrated: access to expert performances and the modelling of processes.
- Provide discussions on conceptual aspects of mathematics, mathematical modelling, and its teaching. This characteristic is consistent with the principles of articulation, reflection, and collaboration proposed by Herrington et al. (2010). The environment allowed the prospective teachers to engage as 'modellers' and later, there was a discussion about their own learning and how it was accomplished.
- Reflect on aspects of the future professional performance in school contexts. It means situations involve the aspects that occur in the classroom reality. These situations also involve an analysis of how mathematical modelling addresses such aspects to allow the development of abilities to anticipate, formulate and influence the problems of social and educational environments, among others. The prospective teachers had to be

involved in critical reading about modelling experiences reported in the literature. Also, they interacted with in-service teachers who offered them workshops during the course. According to Herrington et al. (2010), these actions are consistent with the principles of contexts, (school) authentic tasks, and articulation with modelling practices in schools. Through interaction with in-service teachers, the prospective teachers could talk about classroom management and multiple variables in everyday school by integrating mathematical modelling.

According to these three characteristics, the prospective teachers were involved in four types of authentic tasks: *preparing a lesson plan* (see example and features in Sánchez-Cardona et al., 2021), *developing a modelling project* (See examples and features in Villa-Ochoa et al., 2021), *collaboration and interaction with experts, and solving modelling problems* (see Rendon-Mesa et al., 2016). The first task dealt with the design of a class environment using mathematical modelling. The prospective teachers planned the intervention based on the theoretical references discussed above. This task corresponds with learning and authentic contexts and activities that students recognized as important for teaching mathematics through modelling. The second task dealt with the development of a mathematical modelling project. It allowed prospective teachers to make inroads into modelling experiences, learning and use of techniques to model and recognize aspects involved in this type of activities. In this activity, they had a wide range of theoretical references around mathematical modelling (Rendón-Mesa, 2016).

Collaboration and interaction with experts can be considered authentic (professional) tasks because they promote the exchange of experiences, visions and practices with in-service teachers, researchers and other professionals (Rendón-Mesa, 2016; Rendón-Mesa et al., 2021). Finally, in the modelling problem-solving tasks, prospective teachers analyse research and professional papers and solve the tasks described. They also analysed local experiences about modelling (authentic context and activities) and participated in and solved modelling problems and situations proposed by teacher educators.

Another important aspect of the course design was the authentic environments for learning. These environments were configured according to the teaching-learning trajectory proposed by García et al. (2006). For these authors, prospective teachers should initially attempt each task (individually or in small groups), analyse the mathematical aspects involved and the modelling as a means or resource to develop it. Also, the class promoted the recognition of school conditions which would allow the development of experiences. Subsequently, discussion methods (scientific debates, forums, and discussion groups) were used to think about the role of the teacher in implementing tasks. Possible difficulties that could emerge and how to deal with them in educational institutions were also studied. Finally, discussions and reflections on the possibilities and constraints of modelling described in the task were promoted. This teaching-learning trajectory allowed prospective teachers to have the opportunity to collaborate and reflect on the construction of knowledge.

3.3. Data and Analysis

To identify the opportunities of authentic modelling tasks in prospective teachers' learning, attention was focused on dialogues, actions, and reflections when they addressed each of the tasks in the course. For this purpose, data were collected through different instruments, namely participant observation records, class audios, documents and videos produced by the students, and, through interviews at the end of the course. A thematic analysis of data was conducted (Braun et al., 2019; Braun & Clarke, 2006). This analysis is considered "a method for identifying, analysing and reporting patterns (themes) within data" (Braun & Clarke, 2006, p. 79) and it can be conducted through six phases, namely: Familiarisation, Coding, Theme Development, Reviewing, Defining Themes, and Producing the Report (Braun et al., 2019). Data were organised and coded in two phases.

For the first phase, collected data were

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reviewed by the three researchers, later, they wrote down their impressions on the relevance and sufficiency of the data to answer the research question. Subsequently, a first analysis was carried out, seeking to identify fragments of data that would evidence prospective teachers' learning. An inductive orientation coding was conducted; this orientation, "where the researcher starts the analytic process from the data, working 'bottom-up' to identify meaning without importing ideas" (Braun et al., 2019, p. 853). There were no pre-defined categories, and the analysis was done keeping in mind just the research question. Later, the extracted fragments were analysed and discussed among the researchers, considering the principles of authentic learning (Herrington et al., 2010).

Based on the aforementioned discussion, a new coding system was constructed, and a second deductive coding process was conducted "where the researcher approaches the data with various ideas, concepts, and theories, or even potential codes based on such, which are then explored and tagged within the dataset" (Braun et al., 2019, p. 853). This coding was focused on data that provided information about the nature of the mathematics and the extra-mathematical knowledge while prospective teachers developed their modelling processes. It was also focused on understanding the 'what' and the 'what for' of the mathematical modelling in mathematics teaching, and on the relations to be considered in a school environment.

For the second phase, the research team met for discussion. Based on the theoretical background of authentic learning data, the coding was focused on the data that provided information on how the prospective teachers developed modelling processes, that is, where the phenomenon and the interest emerged, how the study process was developed, what were the purposes and scope of the study, and the implications of its integration into school mathematics.

4. Results

From the two phases of analysis two themes emerged, (i) Learning about Modelling as vehicle and content, and (ii) Learning about the uses of models and learning environments with modelling. The results of these themes are presented in this section.

4.1. Learning about Modelling as Vehicle and Content

As mentioned above, authentic modelling tasks for learning mathematical modelling involve conceptual, cognitive processes of modelling abilities, but also the interest or needs of the subject. Prospective teachers were involved in the design of lesson plans while projecting their role as mathematics teachers through modelling. They also participated in mathematical modelling practices, they developed modelling projects on a topic of their interest, asking and answering mathematically. Tables 1 and 2 present the main features of the lesson plans and the modelling projects.

The prospective teachers showed the importance of providing contexts for mathematical modelling by designing lessons plans and modelling projects. It revealed diverse ways of giving sense to modelling practices. Although most of the lesson plans sought to develop modelling through contexts that included the experience of the students; there were also lesson plans focused on actions to motivate them or focused on contexts that provided mathematics applications in possible scenarios (Table 1).

As evidence, Erika noted in her interview, '*I proposed a class where the students discovered the mathematics application, for example in taxi services. It motivates the students because they can see its application*'. For Erika, the authenticity of the context should create conditions to develop a learning environment that makes students interested and committed to learning mathematics. Erika's statement is a true premise *per se*, and it is not based on her experience or a research report reviewed during the course.

Authentic modelling problems were developed during the course with teacher educators' advice. For example, the study on concentration of

fluoxetine used by patients with depression (see task in Villa-Ochoa et al., 2022), or the study on body mass index and its use by professionals in nutrition (see task in Parra-Zapata et al., 2016). Reflecting on their own experience and their own learning provided first-hand experiences that encouraged the prospective teachers to develop modelling processes in their lesson plans. For example, Carol said, '*I learnt that you could learn mathematics through modelling. Regarding the fluoxetine issue, we saw an exponential function. I took the opportunity to study something similar and I came up with the idea of studying [the composition of] Ritalin. That's why I proposed my class [lesson plan] about it*'. Carlos also stated: '*Working Body Mass Index (BMI) does not allow me to construct the model. However, it allows me to understand not only mathematical concepts of reason, proportion, and the corresponding calculations; but also, how mathematics is present in nutrition and helps us to take care of our health*'.

Table 1 Characteristics of lesson plans

Task type	Purpose	n	Actions	Example of Topics or Phenomena
Authentic	Understand the situation	4	Introducing to the situation, understanding the problem, collecting data, constructing the model, reflecting on results, and communicating results. In some cases, the interaction with an expert is proposed depending on the task type.	Study on the effects of Ritalin.
	Promote a concept or an ability	3	Presetting the situation and data, data interpretation, and completion of procedures to answer the questions. In some cases, reflecting on the process is proposed.	Modelling of a space through software, studying the probability in gender determination.
Construct a fun environment	Use procedures	3	Description of fun environment, actions to respond to the indications.	Games to make measurements.
	Explore concepts	1	A fun environment is presented, and the objects are described.	Fun activities to describe geometric bodies.
Referential	Real situations where mathematics has potential applications.	2	A device and its context of use are presented. Presentation of a simplified mathematical version, and a mathematical idea or explanation.	Taximeter operation for operative calculations in fictitious maps.

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As Carol and Carlos did, other prospective teachers pointed out that after facing tasks like the mentioned above, modelling in the classroom is possible and they recognized some ways to manage the teaching of some mathematical contents through modelling. Table 1 shows that prospective teachers focused their lesson plans on modelling to promote rigid environments where contexts and mathematics contributed to solve stereotyped tasks leading to a procedural treatment of mathematical content (Villa-Ochoa, 2015). Beyond heterogeneous learning in the course, through lesson plans, reflections, and discussions of students, preparation needs of prospective teachers emerged. It imposes new challenges on educational courses and programs.

Other learnings during the course deal with the recognition of different modelling task types. For some participants, modelling does not always have to involve students' daily experience, but they recognize that as teachers, they need to be clear about the purpose of the task and the school conditions. For example, Carlos and Silvia expressed:

Carlos: *authentic modelling tasks are really interesting, for example, that task of the Giant's shoes* (Blum & Borromeo-Ferri, 2009) *I liked it because it shows me a situation that allows the students to propose different strategies*.

Silvia: *I agree with Carlos. I also liked it ["Giant's shoes" task] because the authors showed in many ways what the students could do. That task is not as complex as a project, and it is possible to be adapted taking into account the little time that you have in a class.*

The phase of reflection and discussion (teaching-learning trajectory, García et al., 2006) on solving the modelling tasks and reading articles became a resource to broaden every participant's vision about modelling. It allowed prospective teachers to recognize a diversity of roles, purposes, scope, constraints, and requirements in the different modelling task types. Reflecting and discussing the experience were valued as important to prospective teachers at different moments of the course. For example, Silvia noted, '*I understood some things, but my colleagues helped me to see other possibilities to prepare my classes*'.

The learning about mathematical modelling achieved by prospective teachers was also observed in the development of the modelling project. Each project involved the production of one video, one paper and one dissemination plenary session in front of the group. These projects were proposed as a relevant experience to modelling learning. Its importance was noted when prospective teachers improved the relationship between mathematics and reality.

According to Borba and Villarreal (2005), with the teacher's support, students delimit a situation or phenomenon of interest, formulate questions, design and execute a plan, so that, through mathematics, they solve problematic issues. Prospective teachers as well as professional mathematicians could experience that their object of interest, the plan, and methods had to be adjusted on the conditions that emerged in the study process and recognise the need to work with specialists in other disciplines. According to the nature of the studied phenomenon, these specialists could guide different moments in the modelling process.

Despite the diversity of themes, scopes, contributions, and processes (Table 2), prospective teachers' experiences by developing projects offered opportunities to identify phenomena, recognize magnitudes, and study them mathematically. This highlights the aspects involved in decision-making about the phenomenon to be modelled. Four weeks after the course started, the students had to make an oral presentation to report on possible topics and phenomena to be studied. They presented three or more interesting topics in relation to family economy or environmental local problems, from their labour future or experiences lived throughout their academic career.

For each prospective teacher, choosing a single theme for the project depended on the ease of accessing the data (Alex and Lupe), the impact of results (Leidy), the access to expert advice (José), or the complexity of the mathematical processes that prospective teachers foresaw in the production of models (Erika). For instance, Carol and Jose made their decision based on their visions of the models and corresponding representations. Carol said: '*The truth is, it was very hard at the beginning, I did not know what to*

model, I was very confused with so many forms of modelling, but my partner [José] told me, we can work on the consumption of [plastic] cups, and I said: Is it possible to be modelled?!' Then she added: '*To me, at the beginning, modelling was to get an algorithm or an equation, but in the course, I realised it was not, for example, when we did our project, we did one about the consumption of cups, we constructed a graph and we can call it a model, and I did not know that*' (Carol, interview).

Jose noted, '*When we get one proposal to do a modelling project, without a doubt, we begin to see our needs. We began to see what happened and once we left [the University], we saw several waste from those cups, and we thought about global warming and its effects. Also, contamination [of the air] in Medellín, and the use of water dispensers at the University restrooms. So, we began to see the needs that we had*' (José, interview).

Decisions about the situation to model could depend on the disposition and experience of the participants. The following excerpt is a sample of this diversity:

'*The idea arises from an interest in knowing the influential variables in both the cost and the flight availability because we had flown, but we do not have criteria or sufficient information to make decisions about the place and the method of payment according to our needs*' (Lina and Silvia, video).

Once they understood they had to make a project, Alex, José and their teams began to ask themselves what they could model, and they were aware of what happened every day. Lina, Silvia, Marce and Erika stated that they had many questions about phenomena observed in their daily life. The modelling project was an opportunity to answer those questions. In all cases, the students stated that these types of projects were a new experience and, therefore, the degree of uncertainty was high because they did not have a predetermined path or algorithm to solve the stated purposes. Other characteristics involved in the project development are presented in Table 2.

These modelling project developments were a reference for prospective teachers to look for authentic situations where mathematics is a way to meet the needs that emerge in such situations. Through their reflections on the processes, participants evidenced the recognition of the diverse nature of modelling, the teacher's role by questioning the productions, and the diverse roles assumed by the experts. The prospective teachers recognised the high demands of these processes, but also the learning opportunities they had. Carol, who was already a teacher in an educational institution, said: '*I learn a lot. I had the opportunity to apply the experience in the school [where she works], we applied the situation of Ritalin, it was much more directed, but I had to rely on the chemistry teacher to explain to students about the Ritalin composition, ..., to the students was a nice experience, and to me, it was the best experience during this year*'. She added: *'Every time I look at something, I say here I can do modelling, for example, days ago in a Course, we were asked to do something [study the variation] using a calculator, so I was thinking it was modelling [...] also the weights of new-born children*' (Carol, interview).

As shown in this section, mathematical modelling as a learning object for prospective teachers involves the learning required for making decisions regarding the project, and the recognition of the project contributions and other tasks to school mathematics activities. In the next section, other learning that could occur through the diversity of mathematical modelling approaches is presented.

4.2. Learning about the Uses of Models and Learning Environments with Modelling

This theme presents two relevant findings on the learning of modelling achieved by prospective teachers. The first has to do with the learning concerning the use of models in a specific profession and the second, on some ways of using models in the mathematics class. Both findings emerged from a workshop conducted by a nutritionist and an in-service teacher. Three months before, both had carried out an analysis of the BMI model with 30 fifth-grade children in elementary school. They were invited to the course to talk about this experience.

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Table 2 Other characteristics in the project development

Characteristics	*n*	Appearance in the project	Example
Model construction	3	Previous knowledge of the model assumed in the process	*For the analysis of airfares, we studied airline models.* ***Amadeus*** *provided us with a model, and we studied to understand it.*
	5	The model was constructed by establishing relationships between magnitudes	*Since we could not change the frequency of the toilet paper dispenser, we modified its design to increase the storage capacity in line with the demand at the University.*
Contribution of experts *(Experts contribute in different moments of the Project)*	5	In data collection or delimitation of the problem and variables	*A visit to the planning department of the University to get information about the company, toilet paper dispensers, their use and location.*
	4	In constructing / obtaining the model	*The mechanical engineer helped us with the physical properties of the models like force and friction. In the first model, only the force of gravity makes the toilet paper roll out, but in the modified model, an extra force is created, and it varies according to the angle and material of adaptation. In this sense, it is necessary to use a metal with a smooth facing to have a friction coefficient as low as possible.* (Alex, Lupe and Leidy´s paper)
	2	In the validating model results	*Engineer Higuita confirmed that the production of plastic-coated paper cups spends more than double the amount of water in comparison to the production of polystyrene cups.* (José, video)
Scope of modelling	5	Understand a situation	*This project helped us to understand many aspects behind the consumption of disposable cups and denature some beliefs about the expense of water in relation to washing a personalized cup; however, we did not improve the situation of consumption, because it depends on the awareness of each person.* (José, interview)
	3	Solve a problem by own experience	*With the new dispenser design, we made it possible to meet the demand in the [public] bathrooms of the university building.* (Alex, interview)

The nutritionist attended the class session (2 hours) to analyse the BMI model. In her intervention, she reported theoretical aspects about the origin of the model, uses according to the age and history of the patients. The professional said that it was a measure that provides inputs to classify patients and recommend a treatment. Thereon, prospective teachers proposed case studies to understand the functioning of the model. In this workshop, they asked personal questions, for instance, about the type of foods to consume throughout the day, how to organize a diet, and how to replace a food with a new one.

In another moment of the session, the in-service teacher intervened and showed the class notes produced by her students; she also showed evidence of reflections and uses that they made of the model with her students. The prospective teachers asked how to organize the class, how to prepare the students for a group work, how to assess their learning, how to integrate activities into the curricula. Finally, they asked for the characteristics of the institution where she carried out that activity.

Table 3 shows the prospective teachers' interest about the actions of the expert. On the one hand, they were concerned about the uses and limitations that professionals make of mathematical models in their practices. On the other hand, they were interested in daily factors involved in classroom environments, and in how the teacher designed the environment to allow students to learn mathematics through the study of the BMI model.

Table 3. The main questions of the prospective teachers

	Nutrition Professional	In-service Teacher
Uses of models	How are the results of the model interpreted? Does the use depend on the age and health of the patient? Is there some case in which the use of the model is not possible?	What is the mathematics taught through this model? Could the students know about the uses that professionals make of the models?
Expert roles	What are the data required? What characteristics do we have to consider using the model?	How to teach models? How to organize a team to work with models? How to promote student learning? How to design questions and situations according to the student's grade?
Limitations	Has the model ever provided false results? Does the model always indicate what to do?	How to formulate the curriculum to study the models?

The interaction with experts encouraged the prospective teachers to know about the context and use of mathematical models, both professional users and teachers and students in the classroom. The knowledge of the phenomenon and context was recognised in

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other tasks throughout the course; in particular, prospective teachers were concerned about the development of their modelling project. For example, Silvia and Lina noted, '*[air] ticket costs can be continuously affected by economic factors such as TRM (Market Representative Rate)*'. To make an informed decision on tickets purchase, it should be based on an analysis of how other variables impact prices: availability of flights, rate class, routes, baseline value of ticket, and taxes. Like these participants, Carol and José noted, '*Production of polystyrene plastic cups uses less water than production of plastic-coated paper cups*', besides, *'it is not true that washing your personal cup spends [consume] more water than producing disposable cups*'.

The interaction with the in-service teacher offered possibilities to address the presented challenges by integrating mathematical modelling in their institutions. Beyond this, institutional and cultural conditions constitute a variation in these challenges. Because of that, a solution cannot be transferable to other contexts. The diversity of these challenges led teachers to reflect on the high curricular and contextual demands of mathematical modelling. Studying this diversity exceeded the capacity of this course; therefore, these challenges remain latent for prospective teachers.

Another episode that occurred in the tenth week of the course showed the learning when one participant talked about a successful lesson plan in her school. In her presentation, María stated that she felt questioned about many issues, and she did not know how to respond. In this regard, María expressed, '*Yes, I am teaching a class and one of my students ask me something that I do not know. I would rather say 'I do not know*'. This response caused a reaction from other participants who expressed consternation with her approach but offered personal alternatives: '*I would not do that; on the contrary, I'd use it to consult with the students and learn more*' (Leidy's answer). '*Not knowing is something normal, it can happen to a teacher in a classroom, so I wouldn't tell them lies, I´d just say I don't know and I´m going to search for*' (Lina's answer). '*Look, in the projects, there are many things that neither teachers nor we know, and then we're going to have to find an expert, we cannot tell lies*' (Juan's answer).

In the episode, it was possible to discuss the challenges of doing mathematical modelling, in particular, the need for teachers to recognize that they are not the only ones possessing knowledge and, therefore, the support of other experts in the context of modelling tasks is required.

These episodes provide evidence of the learning in a pedagogical field and of the phenomena and contexts where it is modelled. There is a need to recognize and reflect on the demands of integrating modelling in the classes because it is a complex task.

5. Discussion

Research in mathematical modelling education pointed out to the necessity of providing prospective teachers with opportunities to experience the modelling during their education and include modelling in their plans for future teaching activities (Villarreal et al., 2018). The findings of this study evidence the need of considering at least three aspects when the courses and teacher education programs are designed. These aspects are related to the nature of the experiences proposed to promote learning in prospective teachers, and the corresponding environments where these experiences are carried out.

The first aspect is related to designing and developing authentic modelling tasks for the learning of prospective teachers. Based on the characteristics of authentic learning (Herrington et al., 2010; Strobel et al., 2013), these tasks were conceived not only as situations in which students faced problems and phenomena solved through mathematics but also as experiences and involvement in mathematical modelling projects and interaction with experts (in-service teachers and other professionals). These characteristics offer additional aspects to those proposed in the context of mathematical education.

Different characteristics and contributions of the authentic modelling tasks were identified in this study. The *lesson plans* evidenced that many prospective teachers focused on proposing classroom environments including contexts considered authentic. The purposes and the actions evidenced in the lesson plans were aligned with phases and achievements where modelling was conceived as a process. However, other prospective teachers included tasks in which they presented potential or artificial uses of mathematics, or they focused on promoting procedural abilities. This disparity in results can be interpreted in several ways. On the one hand, previous conditions and experiences could strongly determine visions of the meaning of mathematics and its teaching; on the other hand, the moment in which these lessons were produced (in the eighth week of the semester) may highlight the need to integrate other experiences and reflections on the role of modelling in the daily school life or to design these tasks in a later stage of the course.

The results of this study show that some prospective teachers make a cosmetic use of modelling without representing a transformation in the way of doing modelling. As Kaiser and Schwarz (2010) and Beswick (2011) noted, it is necessary to transcend this type of use and provide authentic tasks as challenges for the students. If these minor uses of contexts in mathematical modelling are overcome, it is possible to contribute to a mathematical understanding integrated with other knowledge specific to contexts and other areas of knowledge (Villa-Ochoa & López, 2011).

A modelling project, as an authentic modelling task for mathematics prospective teachers, can be considered as an opportunity for them to attempt a modelling process by identifying a phenomenon, delimiting a problem, and understanding the importance of interaction with experts and roles of mathematical models. Problems delimited by prospective teachers emerged from the collective negotiation about an interesting topic, as well as the discussion on conditions to develop it. Consistent with the approaches by Herminio and Borba (2010), the interest in a modelling topic is internal and socially shared. The results of this study support the findings of Almeida and Omodei (2022), who found that when problems arise from students' interests, there may not be a predefined set of mathematics to be used in solving them. In this study, mathematics emerged as students defined variables and identified relationships among them. This makes the modelling process more authentic, not only for learning about modelling itself but also by offering opportunities for prospective teachers to understand aspects of its teaching. Additionally, the project development in authentic contexts and the notion of interest allow us to broaden the understanding of how prospective teachers develop a certain sensitivity to identify and study personal and socially relevant situations, that is a 'sense of reality' (Villa-Ochoa & López, 2011). In this study, Carol and José provided evidence of paradigmatic cases in which the project experience contributes to the development of a sense of reality.

Participation in workshops with experts (in-service teachers and other professionals) offers an authentic experience that allows prospective teachers to know first-hand the flexible uses and limitations of mathematical models in both professional and classroom activities. Consequently, this type of interactions makes prospective teachers focus on how and why they use real-life models (e.g. in nutrition) and on factors intervening in a real class; how the teacher can create an environment for the use of mathematical models in school. Results presented in Table 2 evidence expert interventions in different moments of the modelling projects. In a previous study, Rendón-Mesa et al. (2016) showed that interaction with experts provides relevant elements for the development of modelling processes, such as supporting mathematical understanding through the recognition of the use of mathematics in situated contexts. This study supports these ideas by showing that the interaction with an expert (a nutrition professional) enhances knowledge about the professional uses and limitations of mathematical models. However, this study adds additional elements, as interaction with in-service teachers also provides insights into the teaching of modelling. Specifically, this study revealed

that prospective teachers were concerned with recognizing, in real practices, the ways, objectives, and challenges of modelling teaching. Therefore, the authentic modelling tasks offered opportunities for prospective teachers to engage with reliable classroom teaching situations and as credible evidence of what happens in mathematics classes. In this sense, this study suggests new studies to understand how the processes are reorganized depending on the nature of the problem and the phase in which the expert is involved.

This study offers evidence that supports the idea of integrating situations that consider the interest, the phenomena, and a projection of the teaching practices. The development of modelling projects provided a better understanding of the context and phenomenon. Furthermore, interaction with pre-service teachers allowed them to broaden their vision about factors involved when they configure a class environment using modelling. These findings provide empirical evidence about the authentic context, proposed by Barab et al. (2000), Strobel et al. (2013) and Herrington et al. (2010). Beyond this, it is also possible to highlight the personal authenticity and value of authenticity (2013). For example, in modelling projects, phenomena were chosen in accordance with the interest of the prospective teachers. It is an engine that promotes the creation of strategies, collaboration between prospective teachers and with experts, and empowerment to solve the problems.

In the literature, there is a call to overcome stereotyped modelling practices and to attempt situations that reflect the actions of professionals working in the field where the problem comes from (Kaiser & Schwarz, 2010; Niss, 1992). This need has also been reported in other education fields (Herrington et al., 2010; Strobel et al., 2013). In this respect, this article offers empirical evidence to show the attention to this need from other dimensions as a possibility. In other words, as previously reported, some processes developed by prospective teachers are like those processes developed by professionals in applied mathematics. However, they also developed actions like those developed by in-service teachers. In this sense, the authentic experiences analysed in this article allow to recognize that prospective teachers are in contact with mathematical modelling as apprentice of mathematics (modelling as vehicle), apprentice of modelling (modelling as content), and apprentice of pedagogical and didactic aspects. That aspect could be used in future teaching practices.

A third aspect deals with the synergy that should exist in authentic tasks by designing a course for teacher education. In this sense, emphasis is placed on the contributions by the Teaching-Learning Trajectory (García et al., 2006) about the configuration of an environment for learning mathematical modelling. These environments are not a set of short modelling tasks. Instead, the development of tasks is a starting point for academic discussion and the projection of teaching actions; it broadens the understanding of modelling in mathematics education. Carlos and Carol cases are evidence of discussion and reflection on their own modelling experience. It allowed them to organise their lesson plans.

Authentic modelling tasks contributed to articulate the learning of mathematical modelling and its teaching through the Teaching-Learning Trajectory. An articulated knowledge of pedagogy and mathematics results fundamental in the professional development of teachers (Ponte, 2014). In that sense, ask prospective teachers what they learn, how they learn it, how they could teach it, and what difficulties could arise and how to overcome them, were questions that made prospective teachers focus on mathematical contents but also on the means and conditions to achieve these contents.

Discussions and reflections on their own experience provided conditions for Carol and Carlos to apply their lesson plans in the school. As shown above, 'acting as teachers teaching modelling in a real class' helped these two prospective teachers to face particularities as in a real class. Due to the characteristics of the course, the application of lesson plans was not considered as experience. It is a limitation and a challenge to articulate experiences from this course to others.

6. Concluding Remarks

The international literature informs about the need to integrate authenticity in mathematical modelling, and mathematics teacher preparation. In modelling, authenticity provides problems if students need to experience the power of mathematical modelling to understand and solve meaningful real questions. The results of this study extend the existing literature on the issue. It not only seeks to integrate the authenticity of the context and professional practices of mathematicians, but it also integrates other characteristics of authenticity: interaction with experts, and experiences closely related to personal and professional life concerning mathematics, society, and teaching mathematics.

Based on this study, the characteristics of authenticity in mathematical modelling teacher education offer opportunities to:

- Attribute different roles and purposes to the modelling in the mathematical education for prospective teachers and their students.
- Develop modelling processes, propose problems, propose solution strategies, and generate actions to solve them.
- Recognize mathematical modelling as a promoter of the diverse student learning.
- Appreciate the participation of experts in supporting the development of modelling processes.
- Project the characteristics of a learning environment by integrating the modelling into future teaching practices.

Collaboration and communication were also opportunities presented but not broaden in this article. In this sense, it is suggested new studies about how the integration of authenticity in modelling and its teaching aims at the development of these processes. In the environments designed for the learning of the modelling, a synergy among learning mathematics, mathematical modelling, and pedagogy of the modelling was promoted. In this sense, modelling experiences are related to a vision of teacher preparation that integrates conceptual aspects, uses and processes of mathematics with the recognition of the possibilities and limitations offered by the means, resources and environments. Here, other limitations of the study emerged. For example, how to promote other aspects of mathematical modelling, how to recognize other obstacles to integrating modelling, and what strategies to use to overcome them. Further research should address these issues.

Finally, using authentic modelling tasks to learn mathematical modelling by prospective teachers could be worth the effort. Researchers can offer approaches to the multiplicity of tasks that could integrate the authenticity into mathematical modelling, purposes, scopes, and the way of incorporating particularities and needs of every educational system.

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